DISCOVERING THE ART OF MATHEMATICS:
INQUIRY-BASED LEARNING IN MATHEMATICS FOR LIBERAL ARTS

I. INTRODUCTION

Our guiding vision for Discovering the Art of Mathematics is:

> Vision: Mathematics for Liberal Arts students will be actively involved in authentic mathematical experiences that are both challenging and intellectually stimulating, that provide meaningful cognitive and metacognitive gains, and that nurture healthy and informed perceptions of mathematics, mathematical ways of thinking, and the ongoing impact of mathematics not only on STEM fields but also on the liberal arts and humanities.

Discovering the Art of Mathematics (DAoM) meets this challenge directly by developing a foundational library of 10 full-length, stand-alone learning guides, student resources, teacher resources, and assessment tools for Mathematics for Liberal Arts (MLA) courses. Each volume (described more fully in II.A) brings a radically student centered pedagogy (see III and V.B) together with sophisticated mathematical content that embody deep and natural connections to the liberal arts and humanities (see III and V.C).

MLA is an enormous undergraduate audience (see IV and V.A) that is too often served by traditional lecture style courses that are disconnected from the academic interests of its audience (see V.C and V.D). The materials that make up this library provide a unique alternative. Teachers find the materials provide concrete, successfully tested vehicles to help make learning in their classrooms more student-centered. Students find a new world of mathematics, one where they are actively involved, where they are responsible for their learning, where they have been entrusted to investigate real mathematics, and where they find opportunities to bridge the Two Cultures (Snow, 1960; Davis, 1993) divide.

Phase 1 work has helped DAoM grow into a mature project that integrates four TUES program components via the following Type 2 goals (see VI):

1. Successfully complete the DAoM curriculum materials.
2. Develop Teacher Resources Guides and assessment materials for each volume.
3. Disseminate the series and its supporting materials nationally through robust publication.
4. Provide resources for the expanded use of inquiry in general education mathematics courses.
5. Implement a series of 12 professional development "traveling workshops" targeting faculty who are generally without the resources to explore and nurture the use of alternative pedagogies.
6. Create and employ assessment tools that will evaluate the impact of project materials on student achievement and attitudes and may add to the knowledge base on the efficacy of inquiry-based learning in collegiate mathematics.

DAoM is unique, an entire library being built to make the vision above a practical reality. Section V describes key pedagogical and content needs in MLA. With a purposeful focus on the arts and humanities, DAoM directly engages MLA students in bridging the Two Cultures divide. Knowledge from a strong research base has been translated into a practical pedagogy. Section II below provides further details of the compelling **intellectual merit** of work already completed, including six peer-reviewed publications, external review and beta-tests that have uniformly praised the quality and potential impact of the library, and considerable learning gains and affective benefits already seen over 1,000 students who have used DAoM.

The six project goals for Type 2 build naturally on the successes and challenges of our earlier work. Many signs point to the continuing high degree of **intellectual merit** of this work, including the recent invitation to serve as Guest Editors for a special volume on IBL in MLA for the journal *PRIMUS*.

Over 1,300 students will directly benefit from Phase 1 of DAoM. During Type 2 there will be a similarly sized local cohort and upwards of 200 faculty who will be directly impacted, subsequently involving some 4,000 students as a secondary effect. The professional development workshops, offered in conjunction with the American Mathematical Association of Two-Year Colleges, will successfully integrate outreach efforts with the development of teacher materials and assessment tools, recruitment of

reviewers and beta-testers, and dissemination efforts. High-profile, nationwide dissemination through a variety of channels will also help DAoM work have a **Broad Impact**, transforming the way mathematics for liberal arts is taught and learned.

II. DISCOVERING THE ART OF MATHEMATICS – PHASE 1 CONTEXT AND RESULTS

The vision of the 5/15/09 – 5/15/12, Phase 1 project is the same as the proposed Type 2 project. The Phase 1 project was focused on a single project component – Creating Learning Materials and Strategies.

A. Results

The Phase 1 project has demonstrated significant intellectual merit and broad impact.

Creation of a Library of Inquiry-Based Learning (IBL) Guides for Mathematics for Liberal Arts (MLA) – Outstanding progress has been made developing the 10 inquiry-based learning guides and accompanying Student Toolbox that are at the heart of the Discovering Art of Mathematics (DAoM) library. This library is designed to support inquiry-based approaches for MLA and cognate audiences. Each volume is 6-10 chapters, is 100-150 pages, is self-contained and fully indexed, and will provide publication quality curriculum materials for a semester-long MLA course. The guides consist primarily of guided-discovery investigations to nurture students' re-discovery of substantial mathematics. Accompanying text that makes up the remaining third of each guide contains definitions as well as narratives that provide historical, philosophical, artistic, and humanistic context for the mathematical topics under investigation.

As shown in Table 1, over 800 pages of original curriculum materials have been created in conjunction with development, alpha- and beta-testing in 38 sections of Westfield State University's (WSU) MLA course between Fall 2008 and Spring 2012. More than 1,300 students will have used DAoM as their primary curriculum materials by the end of the upcoming spring semester. Additionally, Table 1 details the significant volume of external review and/or external beta-testing that has been completed (as indicated by specific dates) and that is upcoming (as indicated by lack of a specific date).

"Percent complete" in Table 1 does not refer to publication ready, but rather what percentage of the investigations and text have been developed in draft form and typeset. Version stages, described more fully as part of Table 2, are: developmental (0.#), limited external use (alpha = a.# and beta = b.#), full external use (1.#), and dissemination-publication ready (2.#). While some sections of some volumes are in mature stages of development, a central component of Type 2 work is the successful completion of the DAoM library.

	Percent Complete		Current	Latest	# Semesters	External	External
	Investigations	Text	# Pages	Version	Internal Testing	Reviews	Testing
Infinite	100%	100%	105	1.0	5	Karaali, F10	Fox, S12
Geometry	100%	100%	98	b.2	6	Stein, F10	Futamura, F11
Knot Theory	100%	100%	89	b.1	4		
Music/Dance	100%	40%	84	a.3	5	Durfee	Schaffer
Puzzles/Games	100%	50%	148	a.3	8	Ensley	Storm, F10
Number Theory	100%	100%	107	b.1	0	Dudley, F10	Berkove
Patterns	60%	40%	94	a.2	4	Farmer, F11	Fung
Proof,...	40%	40%	69	a.1	4		
Art/Sculpture	30%	30%	50	0.1	1	Loe, F11	
Calculus	30%	10%	20	0.1	1		
Student Toolbox	30%	10%	33	0.1	0		

Table 1 – Individual Volume Status

The vignette (see III below) illustrates project materials in action. Two brief excerpts from individual guides are provided as supplemental documents, and you are encouraged to consider them carefully as the curriculum materials are central components of this project. To see the full depth of the ongoing

curriculum development work, please see the entire manuscripts themselves, each of which is freely available online at http://artofmathematics.westfield.ma.edu .

Scholarly Publications – The scale of the project's curriculum work has moderated publication activity. Nonetheless, the first publication that was a direct result of this project (Fleron, 1999) has been joined by five additional refereed journal publications during Phase 1:

a) "Mathematics and Salsa Dancing," Volker Ecke and Christine von Renesse, *Journal of Mathematics and the Arts,* vol. 5, no. 1, March 2011, pp. 17-28.
b) "Navigating Between the Dimensions," Julian F. Fleron and Volker Ecke, *Mathematics Teacher*, vol. 105, no. 4, November 2011, pp. Cover, 241, 243, 286-292.
c) "Exploring 3D Worlds Using Google SketchUp," Jenny Livingstone and Julian F. Fleron, *Mathematics Teacher*, to appear February 2012.
d) "Radon-Kaczmarz Puzzles: CAT Scans Meet Sudoku," Julian F. Fleron, *Math Horizons*, to appear February 2012.
e) "Musical Palindromes for Liberal Arts Students," Christine von Renesse, *PRIMUS*, forthcoming.

Peer Review – Uniformly positive reports from external beta-testers and reviewers have been encouraging. Notable reviewers include: U. Dudley, K. Ono, G. Andrews, D. Farmer, G. Karaali, and B. Stein. Please see the excerpts from external reports that are included as a supplemental document.

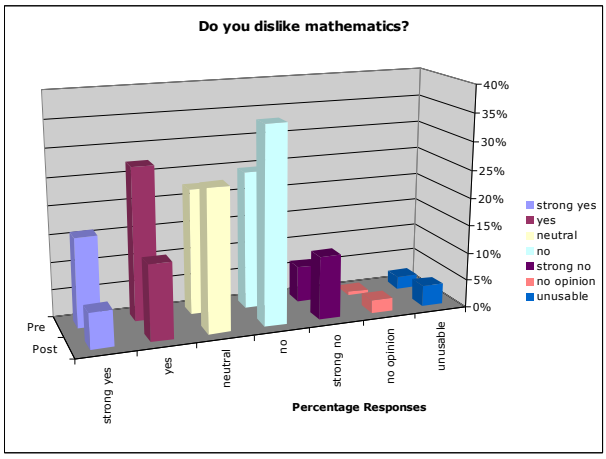
Conference Presentations – Senior personnel have presented results at ten national conferences.

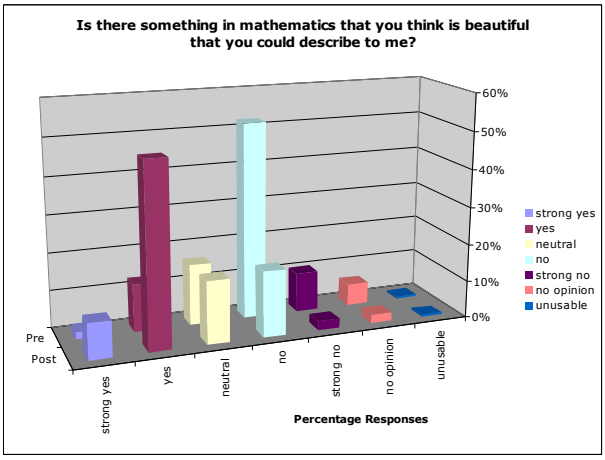
External Contributiors to DAoM – DAoM is seen as a founding library. Original materials from two beta-testers have already been created and significantly more are expected during Type 2.

Evaluation Data – Formative assessment, provided by over 1,300 students, has been critical. Summative evaluation, while not a central project component of Phase 1, has guided the project's vision, project work, and the direction of this proposal.

Summative evaluation comes from many forms. First, initial aggregate institutional data already illustrate substantial impact of DAoM. During the 2008 – 2011 academic years, 2,356 students enrolled in MLA courses at WSU, 933 in sections whose primary curriculum materials were DAoM. The D/F/W rate in these sections was 12.22% while it was nearly double, 24.25%, in non-DAoM sections. 175 students, five full sections, of unsuccessful students would have been successful had the overall rate been 12.22%.

For many years a short, paper survey has been used (n > 500 students). It highlights important changes in student affect from the beginning to the end of the semester, including those illustrated below:





During Phase 1, electronic means have been instituted to collect, store, and analyze expanded survey data (n > 500) and student journals (n > 300) from DAoM students. These data show strong positive results.

The survey instrument and the analysis scales are slightly modified versions of those created by Ethnography & Evaluation Research (E&ER) for the largest study to date on the use of IBL at the collegiate level. (Larsen, et al., 2011) Pre/Post surveys were administered in all DAoM sections over

four semesters, providing 195 matched responses. In each area effects were positive relative to project goals. Many areas showed not only positive, but statistically significant effects: at the < .001 level – *interest*, *enjoyment*, and *interaction*, at the < .01 level – *group work* and *instructor centered*, at the < .05 level *extrinsic goals* and *reasoning*. The changes at the lower significance levels actually mask much more significant changes that fall along gender lines that will be studied as part of Type 2 work.

The magnitude of these effect sizes is comparable to those seen by E&ER. Given the importance of their study, data collection from DAoM may be an important contribution to the research base on IBL.

Journal data were also positive overall. One Pre/Post journal prompt asked students to rate the importance of mathematics in science, humanities, and the arts, explain their thinking, and then provide three examples of mathematics in the world around them. 290 student journals (111 matched responses) were coded to capture the content area and the level of their examples. The percentage of students who gave at least one *advanced example* increased from 57% in the Pre journal to 86% in the Post journal. Another journal assignment asked students to rate aspects of their course experiences. Administered to 238 students in AY2011, over 80% of students report levels of **activity and involvement** in the learning process that are "more" or "much more" than in other mathematics courses. Over 75% of students reported that the amount they have **reflected about how they learn** are "more" or "much more" than in other mathematics courses. Over 68% of students reported that this course has helped **strengthen their reasoning and problem solving abilities** "more" or "much more" than in other mathematics courses.

B. Challenges Uncovered by Phase 1 Work

A number of unexpected challenges arose during Phase 1. First, the strength of the dominant paradigm for MLA that DAoM seeks to confront was underestimated. This had a negative impact on both dissemination and the recruitment of reviewers and beta testers. Second, many challenges arose as the project was scaled up from the proof-of-concept stage. Workload was dramatically underestimated, the required evaluation and assessment resources were not accounted for, and the necessary technical support was not budgeted for. This Type 2 proposal positively embraces these challenges and oversights, believing the positive responses described here will substantially increase the project's merit and impact.

III. VIGNETTE – A GLIMPSE INTO OUR MATHEMATICS FOR LIBERAL ARTS CLASSROOM

Understanding the transformative impact the pedagogical approach of Discovering the Art of Mathematics (DAoM) has on Mathematics for Liberal Arts classrooms is fundamental to the evaluation of this proposal. Since, sadly, you cannot visit actual DAoM classrooms, this vignette brings a DAoM classroom to you. This vignette is loosely transcribed from an actual class. Proposed Type 2 work will include the creation of classroom videos that illustrate pedagogy, exhibit classroom dynamics and serve as professional development tools.

Vignette – It's still five minutes before class starts when I walk into the room. There are two or three students seated at each of the nine tables in the classroom. A few students are sitting idly or chatting while getting organized, but most are already "getting their math on." The direction of their mathematical research is laid out by series of Investigations that make up most of their "textbook" Discovering the Art of Mathematics – Patterns. Ideas, diagrams, data, guesses, patterns, computations, and rationales – their work on these investigations – are recorded in their notebooks. Later solutions, explanations, conjectures, proofs, and mini-essays will emerge from this rough work and will be assessed.

I greet students individually when we make eye contact, put the quote of the day on the board, and tour the room to see how everyone's work is progressing. As I do, the room fills up to its capacity of 35, and by the time class is supposed to "start" every student is doing mathematics. I've said nothing to the class as a whole and given no direction for them to start working, yet each student/group has picked up their mathematical exploration just where they previously left off. It takes only a few classes at the beginning of the semester for the students to become used to being the center of the mathematical experience.

Our most recent foray into the world of patterns is a more strictly mathematical landscape than usual; the prime numbers. We're looking for strings of primes generated by a cylindrical "quadratic number sieve" that each student has constructed out of paper expressly for this purpose.

I approach a group who is a bit behind and now all of its members are focused on our materials. As I expect, they're having trouble deciphering a suite of investigative prompts:

> So what else can we say about the distribution of the primes? Consider the sequence of numbers $10^6!+2$, $10^6!+3$, ..., $10^6!+10^6$.
>
> 46) Explain why the numbers in this sequence are consecutive numbers.
> 47) How many numbers are there in this sequence?
> 48) Show that each of the numbers in this sequence in composite.
> 49) Contrast your answer in 48) with the conclusion of our earlier investigation we called the twin prime conjecture.

I urge them to be a bit more physically active and write things out. Almost immediately one student proclaims, "I get it." She quickly takes on the role of teacher, the group is happily back on their own, and soon they have caught up with the other groups.

I approach another group with communication needs; "Our group was in the library last night and we were having trouble figuring out how to explain this." They are working on Investigations 66) – 69). Sieves in hand, they show me exactly what they had discovered. As they articulate their ideas verbally, it becomes clear to them how they can construct clear, coherent, well-supported descriptions and rationales for the results of their investigation. I have done nothing but stand and listen.

> 66) Show that the first differences of the values of the quadratic $q(n) = n^2 + n + c$ will be the same as those of the function $f(n) = n^2 + n$ and $s(n) = n^2 + n + 17$ we investigated above regardless of the value of c.
> 67) Rotate the sieve so the sieve setting is 17. What do you notice about the entries in the windows? I.e. precisely how do these entries relate to results of earlier investigations?
> 68) Repeat 67) with a sieve setting of 0.
> 69) Find a relationship between entries in the windows for a given sieve setting and the functions q parameterized by the value of c. Describe this relationship precisely.

I continue around the room to a group who I notice is less active than normal and query them about their progress. "We don't know what to do, how to get conjectures for this goal." They point to the prompt:

> 70) Independent Investigation - Understand the ways that functions $q(n) = n^2 + n + c$ generate primes as outputs depending on the value of the parameter c.

I ask them to show me what they've done. They use their sieves to illustrate that they clearly have several well-developed strategies for looking for strings of primes. "Why aren't you using even settings?" I ask. "Because all the results are even. They're not prime." "Shouldn't that be a conjecture? Isn't that part of your understanding of the goal?" I can see the change wash over them as they begin to talk excitedly about other parts of their strategy that quickly become conjectures that are recorded in their notebooks.

As I pass another group one of the students tells me, "Yesterday in my criminal justice class the professor was talking about burden of proof in civil versus criminal trials. That's kind of like all that stuff we've done about inductive and deductive reasoning, right?" The group and I talk about the parallels. As we talk I realize a contentious debate about something mathematical is brewing in a group across the room. I'm not close enough to overhear the details, but I know this is fundamental to real learning. I'll check on them in a bit to see what they have concluded.

Class goes on happily like this for about thirty minutes until I call for a pause. I tell students that, since we've done a "notebook quiz" and an "oral exam" for the last two sections, they will write up complete

results for summative assessment of this section. I show them the award-winning, lead article "Prime Number Patterns" (Granville, 2008) in the April *American Mathematical Monthly* and the cover of the May, 2008 *Notices of the American Mathematical Society*. I'm excited because the former describes tantalizing progress towards solutions of questions we are investigating and the latter shows a two-dimensional Sieve of Eratosthenes that bears striking resemblance to the quadratic number sieve we have built and used for the past few weeks. The students see that the original course materials have given them a context for and connection to these contemporary, high-profile research papers. I'm eager to see their essay responses to summary questions that close this section, which will soon be handed in:

> Essay 1) How does it feel to be working on mathematics fundamentally related to two $1 Million Millennium Prize Problems?
>
> Essay 2) In your opinion, how do you think mathematicians feel about the status of our understanding of the distribution of the primes?
>
> Essay 3) You have read about the importance of primes in data encryption and security. Should this have some impact on mathematicians' efforts to solve the Riemann Hypothesis?
>
> Essay 4) In this section you worked on mathematics that has captivated mathematicians through the ages. You have learned about some of the history and contemporary progress. Compare and contrast mathematicians' progress in this area with the progress of practitioners in specific area in their different field of thought.

The first comment when we get back to "getting our math on" gives me every confidence their responses will be good because one student calls out, "Hey Doc." As I approach she asks, "Is that Wiles guy who solved Fermat's last theorem working on this stuff we're doing too? This is number theory too. He didn't get $1 Million for his problem; I'd be working on this Riemann hypothesis if I were him." The student smiles and her group members nod in agreement.

IV. Inquiry-Based Learning and Mathematics for Liberal Arts Defined

It is hoped the vignette made the following key features nurtured by this project's approach clear:

- The main work of the course, both within and outside of class, is problem solving.
- The majority of class time is spent on student-centered activities.
- The course is driven by a carefully built sequence of investigations that guide rediscovery.
- The teacher's role is decentralized, acting as a coach instead of a knowledge dispenser.
- Students are empowered by playing active roles – determining how class time is spent, initiating communication, and taking responsibility for learning.
- Students use reflection as well as active communication, both verbal and written, to assimilate new modes of thought, new learning strategies, and new mathematical schema.

These characteristics are common to other **Inquiry-Based Learning** (**IBL**) initiatives in post-secondary mathematics (Laursen, et al., 1011) and will serve as an operational definition of IBL for this proposal.

The term **Mathematics for Liberal Arts** (MLA) will refer to general education mathematics courses for students whose major does not require a particular course. MLA courses are typically terminal, introductory level mathematics courses without prerequisites. Course descriptions often talk of broadening students' perceptions and appreciation of mathematics. Typically, MLA is where Visual Arts, History and Government, Religion, Philosophy, Performing Arts, Communication, Foreign Language, Sociology, Music, English, and Literature students have their last formal contact with mathematics.

V. Need for Project and Theoretical Basis for Initiatives

"The vision of the TUES program is excellent STEM education for *all* undergraduate students." (NSF 10-544; IIA; our emphasis.) Mathematics for Liberal Arts (MLA) students form an enormous audience whose needs, described below, are not generally well met. Discovering the Art of Mathematics meets this audience's critical needs by engaging them with radically student-centered pedagogy and curricula that are deeply connected to their interests and chosen fields of study.

A. Enormous Audience

The most recent Conference Board of Mathematics Sciences report certifies the enormity of MLA enrollments. Over 182,000 two- and four-year college students were enrolled in MLA courses in fall 2005, a cohort whose size was exceeded only by students enrolled in calculus, college algebra, and introductory statistics. Moreover, MLA is one of the fastest growing populations, growing at more than three times the rate that overall mathematics enrollments grew in the period 1990-2005. (CBMS, 2007)

B. Pedagogy Challenges

There is a fundamental disconnect between pedagogical theory and practice in MLA. A "Basic Principle" of Crossroads of Mathematics is: "*Mathematics must be taught as a laboratory discipline.* Effective mathematics instruction should involve active student participation." (AMAYTC, 1995) Yet there are only a few, isolated examples of curriculum materials that support active learning. (Jacobs, 1994; Farmer, 1996; Farmer and Stanford, 1995; Kinsey and Moore, 2002) This is particularly troubling as an MLA course "requires more planning and preparation than almost any other mathematics course if it is to be successful" (CUPM, 1983, p. 111) and "the development of new materials based on the standard set forth in this document is essential to lasting reform." (AMAYTC, 1995)

The Boyer Report tells us, "The inquiry-based learning urged in this report requires a profound change in the way undergraduate teaching is structured... Traditional lecturing should not be the dominant mode of instruction... The ideal embodied in this report would turn the prevailing undergraduate culture of receivers into a culture of inquirers, a culture in which faculty, graduate students, and undergraduates share an adventure of discovery." (Boyer Report, 1998, p. 16) Yet "in most institutions they [MLA courses] are given very low priority; they are frequently taught perfunctorily, without a clear set of objectives, by faculty who lack appropriate interest or credentials." (CUPM, 1983, p. 109) More recent statistics suggest little change. In 2005, 79% of MLA courses taught at two-year colleges were taught using the "standard lecture mode." (CBMS, 2007, p. 148)

Discovering the Art of Mathematics is designed precisely to provide a translational bridge between theory and classroom practice. The explicit pedagogy is guided discovery, a descendant of the ancient Socratic approach to teaching, which has a long history. (Hadamard, 1905; Young, 1906; Bronowski, 1956; Moise, 1965; Koestler, 1966; Lakatos, 1976; Freudenthal, 1991) This is a purely practical choice, results through Phase 1 have shown guided discovery to be remarkably effective in building student interest, confidence, and ability levels in MLA.

To help achieve the type of radical pedagogical restructuring that is necessary for MLA the translational bridge between theory and practice that is provided by Discovering the Art of Mathematics is wide. Supported by teacher resources and assessment tools, the curriculum has pedagogical flexibility to support a whole continuum of approaches from some lecturing through even less structured discovery than explicitly appears in the materials. This provides support for a diversity of teaching styles and experience, different institutional audiences, and different departmental requirements. This approach is both informed by and responsive to the challenges of many other active learning traditions, including: active learning in general (NRC, 2000; Donovan, 2005; Postman and Weingartner, 1969; Bickman, 2003; Davidson, 1989; Dubinsky, 1997; Finkel, 2000); the Moore Method (Moise, 1965; Whyburn, 1970; Jones, 1977; Parker, 1990; Mahavier, 1999; Coppin, 2009); inquiry-based learning, in a stricter sense than we have used the general term above (Audet and Jordon, 2005; Lee, 2004); and problem-based learning (Amadore, et al., 2006; Delisle, 1997; Duch, 2001; Schettino, 2003 and 2011).

C. Content Challenges

Quantitative literacy and applications of mathematics have seen renewed attention. Yet instead of being integrated across the curriculum the burden has been left with mathematics teachers. In MLA this has been at the expense of the natural, powerful connections that have long kept mathematics as a liberal art and a cornerstone of higher education because of its fundamental connections to philosophy, the arts, aesthetics, history, humanism, culture, and reasoning. These connections should be a fundamental part of

MLA since this is the last formal mathematical experience for most liberal arts and humanities students. As the Committee on the Undergraduate Program in Mathematics powerfully wrote,

> College students study the best paintings, the most glorious music, the most influential philosophy, and the greatest literature of all time. Mathematics departments can compete on that elevated playing field by offering and making accessible to all students intriguing and powerful mathematical ideas… Indeed, these courses [general education and introductory mathematics courses] should be developed and offered with the philosophy that the mathematical component of every student's education will contain some of the most profound and useful ideas that the student learns in college. (CUPM, 2004, p. 28)

Crossroads in Mathematics reminds us "It is particularly important that liberal arts students understand the impact that mathematics has had on art, history, literature, and many areas of human endeavor." (AMATYC, p. 22) An earlier CUPM panel offered similar guidance, "Students must come to understand the historical and contemporary role of mathematics, and to place the discipline properly in the context of other human intellectual achievement." (CUPM, 1983, p. 110) Unfortunately, topics such as these, which are central to the liberal arts, "are ignored by many authors of mathematics for liberal arts texts. Although mathematics is central to modern science and modern western culture, most texts bypass these important aspects of mathematics." (White, 1998, p. 675) Several groups have been formed explicitly to renew the role of the Arts in STEM suggesting that STEAM, with an A for Arts, is a more appropriate acronym. (STEAM Not STEM and STEM to STEAM) There is even a U.S. House bill encouraging the inclusion of art and design in the Elementary and Secondary Education Act, higher education STEM curricula, and Federal STEM programs. (Langevin, 2011)

Discovering the Art of Mathematics meets these challenges directly. The curriculum is designed around intellectually challenging mathematics with deep and natural connections to the liberal arts and humanities. Again, the goal is translational. The deep connections of mathematics to art, history, culture, humanism, philosophy, and literature are known. This project provides the bridge that allows these connections to be fully incorporated into the mathematical experiences of MLA students for whom they are so very relevant.

D. STEM Awareness and Attitudes

CUPM also specifically highlighted general education courses such as MLA because they "are especially challenging to teach because they serve students with varying preparation and abilities who often come to the courses with a history of negative experiences with mathematics. Perhaps most critical is the fact that these courses affect life-long future workers and citizens." (CUPM, 2004, p. 27)

Rising Above the Gathering Storm (NRC, 2007) and other high-profile public policy documents provide dire warnings about domestic STEM shortages. While there are many factors, the health of STEM awareness and attitudes certainly must be considered as causes. MLA provides an awesome opportunity to address STEM awareness and attitudes among a particularly challenged cohort:

> Liberal arts education, for a large percentage of the college educated population, is a rigorous, disciplined encounter with the best elements of man's history and culture. The major clientele of the mathematics appreciation courses are liberal arts students, and it is from their ranks that many of society's leaders will emerge. (CUPM, 1983, p. 110)

Discovering the Art of Mathematics accepts the challenge of developing "citizens knowledgeable about STEM and how it relates to their lives." (NSF 10-544; I)

VI. DISCOVERING THE ART OF MATHEMATICS – PROPOSED TYPE 2 WORK

All results from Phase 1 suggest that the vision of Discovering the Art of Mathematics (DAoM) is attainable; this project enables Mathematics for Liberal Arts (MLA) students to have profoundly positive mathematical experiences through the use of Inquiry-Based Learning (IBL). What makes this possible is the availability of high quality curriculum materials whose content and pedagogy are effective and accessible. This Type 2 proposal seeks support to complete the curriculum work initiated as part of Phase

1 and to develop teacher resources and professional development workshops that enable a large cohort of teachers to similarly transform the experiences of students in their MLA classes. Development of materials to date and positive assessment/evaluation of these materials (see II.A) should provide sufficient evidence that "further investment in the new materials or approaches is justified." (NSF10-544; II.A)

A. Type 2 Goals

This Type 2 project will address four TUES project components:

- Creating Learning Materials and Strategies – See Goals #1 and #2 below.
- Implementing New Instructional Strategies – See Goals #1, #2, #3, #4, and #5 below.
- Developing Faculty Expertise – See Goals #3, #4, and #5 below.
- Assessing and Evaluating Student Achievement – See Goal #6 below.

For each goal below, **those passages bolded are the measurable outcomes for that goal.**

Goal #1 – Complete Discovering the Art of Mathematics Curriculum Materials

Research, development, writing, classroom testing, external review, external beta testing and revision will continue as in Phase 1. At the conclusion of the proposed Type 2 project **this will result in a completed library of 10 full-length, self-contained, fully indexed inquiry-based learning guides (each 100-150 pages; sufficient curriculum materials for a semester-long course) that are publication ready.**

As shown in Table 1, the major research, development, and draft writing of the volumes will be nearly 2/3 complete at the conclusion of Phase 1. While this is great progress, DAoM is at a critical juncture. As shown in Table 1 and Table 2, significant internal testing, external review and beta-testing, and editing in response to these activities are required for even the most mature volumes to be publication ready.

Phase 1 classroom experience has shown that interactive java scripts and applets, image galleries, online resource lists, and videos would greatly enhance student exploration. **These tools and electronic resources will be developed and incorporated into the curriculum materials.** As described in the budget justification, development will be by senior personnel and implementation by a hired programmer.

Goal #2 – Develop Teacher Resources for Each of the Discovering the Art of Mathematics Volumes

Extensive teacher materials that support each of the volume in the library will be created. Originally envisioned as briefer supplements containing selected resources and selected solutions, Phase 1 work has shown that teachers who are novices to IBL require more extensive supporting materials.

Teacher materials will include more extensive **solution sets** than initially expected, **content-specific pedagogical instructions** for each volume, a repertoire of **skills that help supporting IBL in the classroom**, **detailed descriptions of alternative organization of the topics**, **substantial classroom assessment tools**, and a **collection of classroom videos** (described more fully below). The ultimate content and structure of the teacher materials will be developed in conjunction with work with teachers during professional development, external review and external beta-testing.

Goal #3 – Broad, Vibrant Dissemination of Curriculum Materials and Teacher Resource Materials

Originally, dissemination of the DAoM library was envisioned as print-only. For potential print dissemination senior personnel have been in regular communication with editors and staff of the publication divisions at the American Mathematical Society and Mathematical Association of America as well as the Editors of the *Journal of Inquiry-Based Learning (JIBL)*. This communication will continue. Letters of support from each are included as supplementary documents.

The publishing industry is in great flux and it is unlikely that the entire series could be disseminated as bound, print volumes, for cost and logistical reasons if nothing else. Hence, electronic media will play an important role in dissemination efforts, providing many exciting opportunities, including:

- Providing an alternative to 1000+ page texts, of which only 100-200 pages are used, and which pass on financial costs to students and environmental costs to Earth.

- Enabling DAoM curriculum materials to be used modularly. For example, sections from many different volumes could be used for a course, just a few sections used to supplement a course, or a few sections used to help a teacher begin to experiment with IBL.
- Materials can be shared to be adapted to suit individual needs – perhaps by sharing the material's LaTeX code under the aegis of creative commons licensing.
- Providing the opportunity for online student resources – interactive java scripts, applets, image galleries, and videos – to be fully integrated with the curriculum materials.

The DAoM library will continue to be developed in ways that provide maximum flexibility to respond positively to the fluid nature of the publishing industry. As described in the data management section, LaTeX is used for all typesetting, amsbook is the style, and all online materials are created to work across the major platforms. Senior personnel are experienced with wikis, asynchronous virtual learning environments like WebCT/Blackboard, and several synchronous virtual learning environments. Discussions with organizers of the Electronic Bookshelf through the Center for Mathematics and Quantitative Information and the Open Textbook Initiative at the American Institute of Mathematics are already underway. Other online repositories like Math DL and the Connexions Project at Rice University will also be considered.

DAoM has already inspired users to create materials of their own. **Dissemination will include selected materials that have been developed by reviewers, beta-testers, and professional development workshops participants.** The expansion of DAoM into a larger, peer reviewed library consisting of materials submitted through a general call for papers is beyond the scope of Type 2. This may, however, be part of a Type 3 follow-up proposal if there is sufficient call for such an expanded library.

Goal #4 – Provide Substantial Resources for the Expanded Use of IBL in General Education Courses

DAoM has MLA as its primary audience. Yet there is non-trivial content overlap with many general education mathematics courses, courses for future elementary school teachers, and specialized secondary school audiences. The social, developmental, attitudinal, and – most importantly – pedagogical needs of these more general audiences are deeply related and DAoM is valuable in these cognate settings.

An online Topic Index linking content topics typical to general education mathematics courses to specific DAoM sections is being created. This will expand the cohort of teachers who use DAoM by providing a simple way to find sections of DAoM materials that are appropriate for courses for which an entire volume may not be appropriate. (A draft version with over 50 content topics is available online at http://artofmathematics.wsc.ma.edu/TopicMatrix.html .)

Original scholarly work on the use of IBL in general education courses will be nationally distributed. As noted, five peer-reviewed publications have been completed during Phase 1. Additionally, an invitation from the Editor of the journal *PRIMUS* to serve as Guest Editors of a special issue dedicated to the use of IBL in MLA has been accepted. (See supplemental documents for letter.)

Senior personnel will continue to offer a robust schedule of presentations. As in Phase 1, this will include presentations at national meetings. In the role of Guest Editors for *PRIMUS*, senior project personnel have already filed an application to organize an MAA Contributed Paper Session at the 2013 Joint Mathematics Meetings.

Videotapes of IBL in action will be produced. Sixteen 5+ minute video segments highlighting student interaction, student-teacher interaction, and classroom discussion will be shared online to help teachers, workshop participants and conference participants become more familiar with the use IBL.

Provide expanded awareness of the benefits of IBL General Education mathematics courses. Senior personnel are active participants in many IBL groups: The Legacy of R.L. Moore, *Journal of Inquiry-Based Learning*, Academy of Inquiry-Based Learning (where each of us is a mentor), the Elementary Mathematics Pre-Service Project, and Project PRIME. Letters of support are included as supplemental documents. A central focus of a follow-up Type 3 TUES proposal may be the formation of an umbrella organization that unites the efforts of these different groups.

Goal #5 – Implement Traveling Professional Development Workshops for Faculty to Incorporate IBL in General Education Mathematics Courses

In 2005, 55% of courses for Mathematics for Liberal Arts (MLA) students at four-year colleges were taught by faculty who are neither tenured, nor tenure track. (CUPM, 2007) These faculty, as well as many community college faculty that teach MLA, generally have less time, less support, and fewer opportunities for meaningful professional development. Thus, traditional avenues will not reach those who typically teach MLA. To reach this "diverse group of faculty so that new materials and teaching strategies can be widely implemented" (NSF10-544; II.A) and to help spur the development of active communities of IBL practitioners, **12 on-site professional development workshops providing intensive experiences in the use of IBL in teaching general education mathematics will be implemented.**

Three successful pilot workshops have been held, informing the workshop structure proposed below. Future workshops will be affiliated with the American Mathematical Association of Two-Year Colleges' "Traveling Workshop" series. (See supplemental documents for a letter of support from AMATYC.)

The central activities of these traveling professional development workshops are:

i. A group of 8-12 mathematics faculty from a region, including both full- and part-time faculty from colleges, universities, community colleges, and secondary schools, is formed.
ii. Teams of 2 or 3 members will take part in advance activities in preparation for the workshop.
iii. A two-day, on-site professional development workshop will be held. Workshops will be run by two senior project personnel, with local arrangements (food, rooms, times) coordinated by a local volunteer. Materials from DAoM will serve as content vehicles for the workshop. Particular choices of content are will be determined by local needs and interests.
iv. Responding to their experiences, participants will incorporate IBL lessons in their classrooms.

These workshops will not only reach broader audiences but they will also be more cost-effective than bringing participants together from afar. Additionally, they are responsive to key obstacles in the implementation of new pedagogy: "professional development is more efficient when it is situated in practice" (Blanton, 2009) and "change likely will not take place unless faculty members work with their colleagues to negotiate and create common understandings related to reform." (Sunal, et al., 2001)

For these workshops the goals and related measurable outcomes are as follows:

a. Participants will experience IBL in mathematics as both students and teachers. As preliminary activities participants will watch videotapes illustrating IBL in action and will observe one another's classes. At the workshop participants will observe workshop leaders modeling IBL with local students. Afterwards participants become the students themselves and see IBL from the student perspective, challenging them to reflect on alternative views of teaching and learning.
b. Participants will learn about the resources available to support the use of IBL in their classrooms.
c. Participants will develop and implement an IBL lesson in their classroom.
d. Workshops will nurture the development of a community of IBL practitioners.

In addition to the external professional development goals, these traveling workshops have natural, symbiotic connections to other essential components of the project. Namely, they will 1) help broaden dissemination efforts, 2) aid in the recruitment of beta-testers and reviewers (three pilot workshops have already yielded 4 scheduled beta-testers), and, 3) provide an essential opportunity to communicate with prospective adopters about their students' needs, the nature of faculty curriculum needs, and the types of accompanying teacher material and assessment tools that they would find useful.

Goal #6 - Assess and Evaluate the Impact of DAoM on Student Achievement and Attitudes

With extensive classroom development and testing throughout Phase 1, formative assessment was significant. **Formative assessment will continue to play a critical role in Type 2.**

Phase 1 experiences have helped in the development of original tools that will enable multi-institutional collection of data that measure the effectiveness of DAoM and, more broadly, the use of inquiry-based

learning. The following tools will be implemented on a regular basis locally, externally by beta-testers and on a volunteer basis by other users of DAoM.

An original online survey will be developed and implemented to help measure student confidence levels, interest levels, views of mathematics, and engagement in the learning process. This survey will be adapted from the E&ER survey described above. If larger scale results continue the trends seen in initial data the results will help add to the research base on the efficacy of inquiry-based learning.

A content independent achievement test that measures student progress in reasoning, problem solving, and mathematical ways of thinking will be developed and implemented. Available instruments, such as the NSF-supported Critical Thinking Assessment Test that was used in Phase 1 on a pilot basis, are not sufficiently compatible with project goals to provide meaningful data.

B. Timeline

The creative process that will guide work on curriculum, student, and teacher materials will proceed in stages similar to those that succesfully guided Phase 1. The curriculum development stages are:

- Background Research and Material Development (**Dev**; Versions **0.#**) – Extensive background research on both content and pedagogy informs choices of topics, themes, and investigative vehicles. Draft modules are classroom tested before being more fully developed.
- Alpha Version Testing and Editing (**aTest**, **aEdit**; Versions **a.#**) – First internal classroom testing of nascent materials, with materials developed and heavily adapted in real time in response to formative and summative assessment of the materials by students.
- Beta Version Testing and Editing (**bTest**, **bEdit**; Verions **b.#**) – The learning guide can be distributed at the outset of the course and serve as material for the semester, investigations are fully developed and accompanying text exists in at least outline form. In addition to regular internal testing, there is formative external review and isolated external testing.
- External Review, External Testing, and Subsequent Editing (**xReTe**, **xEdit**; Versions **1.#**) – Beta version of learning guide is at sufficient stage for summative external review, regular external classroom testing, and subsequent editing.
- Final Revisions, Indexing, and Copy Editing (**CEdit**; Versions **2.#**) – Final stages completed in preparation for publication.

Teacher materials will be developed in conjunction with the professional development workshops, external review, and external beta testing. This will require fewer, but analogous, stages: **Dev → xReTe → xEdit → CEdit**. The proposed timeline for project work on Goals #1 and #2 is then given in Table 2.

Work on Goals #3 - #6 will proceed linearly throughout the period of Type 2 work with benchmarks as given in the Evaluation section VI.D below.

	Jan 2012 - May 2013	May 2013 - May 2014	May 2014 - May 2015	May 2015 - May 2016
Infinite	bEdit, xReTe; TM - Dev	xEdit; TM - xReTe	CEdit; TM- xEdit, CEdit	
Geometry	bTest, bEdit, xReTe; TM - Dev	xEdit; TM - xReTe	CEdit; TM - xEdit, CEdit	
Knot Theory	bTest, bEdit, xReTe	xEdit; TM - Dev	CEdit; TM - xReTe	TM - xEdit, CEdit
Music/Dance	bTest, bEdit	xReTe, xEdit; TM - Dev	CEdit; TM - xReTe	TM - xEdit, CEdit
Puzzles/Games	bTest, bEdit	xReTe, xEdit; TM - Dev	CEdit; TM - xReTe	TM - xEdit, CEdit
Number Theory	aEdit, bTest	bEdit, xReTe	xEdit; TM - Dev, xReTe	CEdit; TM - xEdit, CEdit
Patterns	Dev, aTest	aEdit, bTest	bEdit, xReTe; TM - Dev	xEdit, CEdit; TM - xReTe, xEdit, CEdit
Proof,...	Dev, aTest, aEdit	bTest, bEdit	xReTe, xEdit; TM - Dev	CEdit ; TM - xReTe, xEdit, CEdit
Art/Sculpture	Dev	aTest, aEdit	bTest, bEdit, xReTe; TM - Dev	xEdit, CEdit; TM - xReTe, xEdit, CEdit
Calculus	Dev, aTest	aEdit, bTest, bEdit	xReTe; TM - Dev, xReTe	xEdit, CEdit; TM - xEdit, CEdit
Student Toolbox	Dev	Dev, aTest, aEdit	bTest, bEdit	XReTe, xEdit, Cedit

Table 2 – Timeline for Curriculum Materials and Teacher Materials

C. Exceptional Qualifications, Opportunity, and Resources

The four senior personnel are well-positioned to meet the pedagogical, mathematical, teaching, scholarly, writing, and dissemination challenges of this project. Each has a Ph.D. in Mathematics, has substantial background in pedagogy, mathematics education, and the history and philosophy of mathematics, has deep experience teaching diverse audiences, and is a practicing artist. As Phase 1 illustrates, this team has the passion, intellectual interest, and commitment to make this project's vision a reality.

Distribution of project initiatives among the senior project personnel are described in the budget narrative in conjunction with workload, release time, and summer stipends details.

Westfield State University provides an excellent environment for the research, development, and local field testing of the curriculum materials that will make up this library. It is an appropriate place for the genesis of fundamental changes to MLA as Westfield State was founded as a "People's College" by Horace Mann. This institution continues to have a rich Liberal Arts tradition. The Common Core of Studies requires each student to take two mathematics courses. Letters of support from the President and Academic Vice President are provided as supplemental documents.

Westfield State will continue to strongly support this project through Type 2. The Office of Academic Affairs has recommended a sharply reduced rate for indirect costs and will absorb all fringe benefit costs. The University is also providing sizable institutional resources to address project needs beyond the scope of the project budget. As described in the budget narrative, the University will provide dedicated assignments of a project Webmaster, a Hardware Technician, an Indexer/Copyrighter and Public Relations Liaisons and will purchase project-dedicated computer hardware.

The Department of Mathematics, with 12 full-time tenure/ tenure-track faculty, has a mission compatible with this project. It provides "its students… opportunities to experience the wonders of mathematics themselves... The Mathematics faculty is proud of their commitment to creative classroom instruction and active learning... Many of these pedagogies, by requiring that students work together, help to create an inclusive, interactive learning environment where students develop important communication and interpersonal relationship skills." As many as 24 sections of MLA courses are offered each academic year providing ample access to appropriate students and classes for development, testing, and evaluation. The Department Chair, whose letter of support is included in the supplemental documents, is a strong supporter of this project.

An External Advisory Board of nationally recognized experts has been established. This group will meet with the senior project personnel within six months of the beginning of the Type 2 project. Senior project personnel will communicate regularly with the Advisory Board regarding issues in their respective areas of expertise. Additionally, the full board will meet with the senior project personnel once a year to provide guidance, feedback, and perspective on the project's direction and implementation. This board is:

- David Farmer, Director of Programming, American Institute of Mathematics, Ph.D. in Mathematics, Oklahoma State University.
- Jere Confrey, J.D. Moore Distinguished Professor of Mathematics Education, North Carolina State University, Ph.D. in Mathematics Education and Research Methodology, Cornell University.
- Sandra Laursen, Co-Director, Ethnography & Evaluation Research, University of Colorado at Boulder, Ph.D. in Chemistry, University of California, Berkeley.
- Carmel Schettino, Ph.D. Candidate in Mathematics Education at State University of New York at Albany; Ten years as Department Chair, Mathematics Instructor, and Coordinator of Innovative Classroom Practice at the Emma Willard School in Troy, NY.
- Deborah Schifter, Principal Research Scientist at the Education Development Center (EDC), Ph.D. in Psychology, University of Massachusetts Amherst.
- Dorothy Wallace, Professor of Mathematics, Dartmouth College, Ph.D. in Mathematics, University of California, San Diego.

Letters of support from each of the advisory board members are included as supplemental documents.

D. Assessment and Evaluation

Evaluation of this project will be conducted by the SageFox Consulting Group of Amherst, MA under the leadership of Drs. Kenneth Rath and Andrew Habana Hafner. They are the evaluators for many NSF-funded projects with higher education institutions nation-wide, including a number of on-going teacher professional development initiatives at UMass Amherst's STEM Ed Institute and Yale University's Peabody Museum. They will focus on tracking progress toward meeting project goals, providing formative feedback, and providing summative information about project effectiveness. The table below links outcomes and project goals and provides evaluation activities that will be used to assess these outcomes. Explanations of the specific evaluation activities follow.

Goal	Outcomes/Benchmarks	Evaluation Activities
1. Complete DAoM curriculum materials.	All curricular materials will be completed by 15 May, 2016. As shown in Table 2, sans copy editing, 5 volumes will be completed by the end of year 2 and 2 more by the end of year 3.	Progress tracking
2. Develop teacher resources for each of the DAoM volumes.	All teacher resources will be completed by 15 May, 2016. As shown in Table 2, 2 volumes will be developed by the end of year 1, 3 more by the end of year 2, and 3 more by the end of year 3.	Progress tracking
3. Broad, vibrant dissemination of curriculum materials and teacher resource materials.	Curriculum materials and teacher resources will continue to be made available online through the life of the grant. Dissemination options will be actively investigated throughout the grant period and a formal plan for final dissemination will be in place no later than the end of the grant period.	Progress tracking
4. Provide substantial resources for the expanded use of IBL in general education courses.	The topic index will be completed and posted by 15 May, 2013 and will be routinely updated thereafter. At least one scholarly article or four professional presentations will be produced each year. The videotape library will be started in Fall, 2012 and will be added to each semester over the life of the grant.	Progress tracking
5. Implement traveling professional development workshops for faculty to incorporate IBL in general education mathematics courses.	Workshops will be offered three times a year with at least 8-12 participants in each workshop who will implement IBL in their respective teaching, provide feedback, and contribute to an IBL community of practice.	Progress tracking Pre-/Post-workshop instructor surveys Follow-up instructor surveys Instructor interviews
6. Assess and evaluate the impact of DAoM on student achievement and attitudes.	Student confidence, interest, and views of mathematics, and engagement in learning process will increase in IBL classes to a greater extent than in non-IBL classes. Student mathematical reasoning, problem solving, and mathematical ways of thinking will increase in IBL classes to a greater extent than in non-IBL classes. Students will demonstrate substantial learning of specific mathematical content knowledge at a level comparable to or higher than that in non-IBL classes. Students will be more likely to succeed in the IBL classes (i.e. lower D/F/W rates) than in non-IBL classes.	Student surveys Tests of mathematical thinking and content knowledge Grade comparison between IBL and non-IBL classes

Progress Tracking – The extent to which the project is meeting its progress benchmarks will be tracked on an ongoing basis and matched against the initial plan. The evaluators will meet regularly with senior project personnel to understand both positive and negative deviations from the plan.

Instructor Surveys – All professional development participants will be asked to take three surveys. The first, prior to the workshop, will ask about expectations of, interest in and comfort with IBL. The second, after the workshop, will ask for feedback on the experience and how it affected them. The third, a year later, will examine the long-term impact of participation.

Instructor Interviews – All instructors who continue their involvement with the project for more than a semester after attending a workshop will be invited for interviews. The interviews will be used to more completely understand how the curricula are being used within the context of their institutions, how the students have responded to them, and to what extent other instructors have become interested in IBL.

Student Surveys – All instructors will be asked to give their students surveys examining their confidence, interest, and views regarding mathematics and their engagement in the learning process at both the beginning and end of their IBL classes. SageFox will use their extensive experience to guide the adaptation of the E&ER tools, described in section II.A and VI.A, which have already been used in Phase 1. Refined instruments will be closely examined for reliability and validity under controlled conditions with volunteers at Westfield State University (WSU) before being widely used.

In addition, some participants will be asked to administer the same surveys in non-IBL classes, either the same class taught before the introduction of IBL or non-IBL classes with comparable populations. These surveys of non-IBL classes will provide a measure of growth in student interest and attitude in more traditional mathematics courses that will be compared to growth of students in IBL-oriented courses.

Student Tests – Student performance will be examined on two sets of tests. The first will be course non-specific and examine reasoning, problem solving, and mathematical ways of thinking. All participants will be asked to use this test once it has been developed and examined with volunteers at WSU in regard to its reliability and validity. The second will be tests of content specific to the courses, which may be drawn from the assessment materials created by the project team or developed by the participating instructors. In each case, use will proceed in the same fashion as the student surveys, with the tests given at the beginning and end of the year and with a corresponding group of non-IBL students. Changes in performance will be examined within the IBL classrooms and compared to that from non-IBL classrooms.

Grade Comparisons – Institutional data at WSU will continue to be studied in aggregate. Externally, course grades voluntarily supplied by participants will compare classes where IBL was used to those taught by the same instructor, generally in previous years, where it was not used.

Dissemination and Reporting – SageFox will provide the project team with regular reports on the activities undertaken each semester, discussing the findings and implications in their regular meetings with the project team. They will also write the evaluation portion of the annual and final reports for NSF and provide assistance in the production of articles and presentations, as relevant.

VII. CONCLUSION

The vision opening this proposal is grand. When first posed, could you imagine how it could be fulfilled? The vignette provides a glimpse of what reaching this vision looks like. This illustration is not an example, but rather the norm for 1,300 Mathematics for Liberal Arts students who have learned from Discovering the Art of Mathematics. DAoM has entrusted these students with authentic mathematics that they have explored deeply through action, as they would other arts, and which they have investigated to discover fundamental connections between mathematics and the arts and humanities. The learning gains and affective benefits for these students have been considerable.

The value added from Type 2 work on DAoM will be greater, multiplying this vision, and its impacts, across many classrooms at many institutions. The completed DAoM library will supply a broad, permanent base of inquiry-based learning curricula with deep connections to the arts and humanities. This library will also provide extensive teacher resources to enable broader and more effective use. Dissemination, presentations and traveling workshops will actively recruit across the entire spectrum of those that teach Mathematics for Liberal Arts and other general education mathematics courses.

Discovering the Art of Mathematics will fundamentally transform the infrastructure that supports the teaching and learning of Mathematics for Liberal Arts.