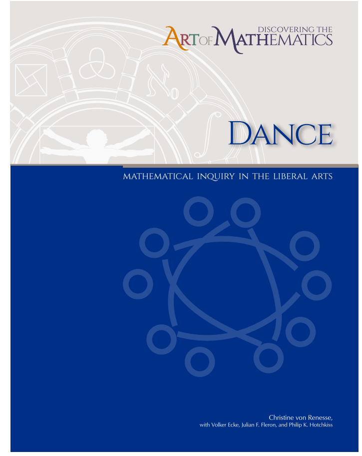

The following is excerpted from:

Discovering the Art of Mathematics: Dance

By Christine von Renesse with Julian F. Fleron, Philip K. Hotchkiss and Volker Ecke

As with all of our learning guides, this book is freely available online at

`http://www.artofmathematics.org/books/`

Discovering the Art of Mathematics (DAoM) is an NSF supported project that supports inquiry-based learning (IBL) approaches for mathematics for liberal arts (MLA) courses.

The DAoM curriculum consists of a library of 11 inquiry-based learning guides. Each volume is built around deep mathematical topics and provides materials which can be used as content for a semester-long, themed course. These materials replace the typical lecture dynamic by being built on inquiry-based investigations, tasks, experiments, constructions, data collection and discussions.

DAoM also provides a wealth of resources for mathematics faculty to help transform their courses. Extensive online resources include volume specific teacher notes and sample solutions, classroom videos of IBL in action, sample student work, regular blogs about teaching using IBL and a regular newsletter. Opportunities for supported reviewing and beta testing are also available.

For departments interested in IBL, DAoM offers traveling professional development workshops.

Full information about the *Discovering the Art of Mathematics* project is available at

`http://www.artofmathematics.org`

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Use the excerpt below to explore for yourself how our materials can engage students in mathematical inquiry.

Dancing Symmetry

Imagine you are standing in front of a mirror.

1. If you move your left arm, which arm is your mirror image going to move?

2. What happens if you move your left leg?

3. And how about turning to the right (clockwise, as viewed form above), away from the mirror. Which way does your mirror image turn?

With a partner, explore this connection: One person is the active person while the other person is the mirror image who is permanently mirroring the moves. Tape the mirror line on the floor so you don't forget where the mirror is. Be creative as the active person, you can move in any way you want, except moving the mirror line itself. Write about your experience. What did you notice? What was easy, what was difficult?

Imagine the following situation: Both dancers face each other across the mirror line and lift just their left arm.

4. Why is the above situation not a mirroring situation? Explain in detail.

We know that we cannot use reflectional symmetry to describe the above position in which both dancers stand facing each other with just their left arms lifted. But clearly it looks and feels symmetric!

5. Think about the two dancers that face each other and both lift their left hand. Imagine you could pick up one person and move it around where ever you wanted. How would you move the person to match exactly with the other person? Act out the movement and describe or draw the process precisely. What would you call this movement?

[Some additional material omitted...] We call this kind of symmetry *rotational symmetry*.

6. Can you imagine why we call it rotational? What is being rotated?
7. And around which point do we rotate?
8. By how many degrees do we rotate?

Now that you know about two kinds of symmetry, we can practice using both. Start with reflectional symmetry, agreeing on a place for the mirror. After creating interesting movements for some time, the leader says "switch."* Now the follower has to follow in rotational symmetry. But there is a problem: not in all positions can you switch smoothly between symmetries, meaning you don't have to quickly adjust your position.

9. Find a position in which you can not switch smoothly from reflectional to rotational symmetry. Explain why.
10. Find several positions in which you can switch from reflectional to rotational symmetry. Draw the corresponding pictures.
11. Describe **all** positions in which you can switch from reflectional to rotational symmetry. This is your *conjecture*.

If we want to be precise and prove a conjecture in mathematics it is helpful to have precise language for the definitions and terms we are using.

12. What do you think: where do definitions in mathematics come from? Who creates them and who decides which ones to use?
13. Is it ok for you to just invent something and call it a definition? Why or why not?

Now you are ready for your first proof†:

14. Describe **all** positions in which you can switch from reflectional to rotational symmetry. Justify that you can actually use the positions you found to switch between symmetries. Explain how you can be sure that you found **all** of the positions.

*This exercise is inspired by www.mathdance.org
†If you want to know more about proofs look at the guide *Discovering the Art of Mathematics: Student Toolbox*