DERIVATIVE RULES

Power rule: $(x^n)' = nx^{n-1}$.

Multiplied Constants Rule: $(cf(x))' = cf'(c)$.

1. Explain the Multiplied Constants Rule in your own words.
2. What is the derivative of $4x^2$?
3. What is the derivative of $0.5x^{\frac{1}{2}}$?

Sum Rule: $(f(x) + g(x))' = f'(x) + g'(x)$.

Take the derivative of the following functions:

1. $5x^4 + 16x^2 - 10x$
2. $\frac{1}{x^4} + 18x^{52}$
3. $40x^2 + 76$
4. Explain why it makes sense to you that the derivative of $14x - 5$ is 14.
5. Explain why it makes sense to you that the derivative of 5 is 0.

Exponential Function with base e**:** $(e^x)' = e^x$.

1. Explain in your own words why e is a special number.
2. Draw the graph of e^x and convince yourself that $(e^x)' = e^x$. Explain your strategy.
3. Find the derivative of $3e^x + x^7$.
4. Find the derivative of $e^x - 17e^x$.

Product Rule: $(f(x) \cdot g(x))' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$.

1. Find the derivative of $e^x \cdot x^2$.
2. Find the derivative of $x^2 \cdot x^3$ using the product rule.
3. Find the derivative of $x^2 \cdot x^3$ using first some algebra and then the power rule. Should your answer agree with the product rule result? Why? Does it agree?
4. Many students **incorrectly** believe that $(f \cdot g)'(x) = f'(x) \cdot g'(x)$. Give an argument why this cannot be true.
5. Find the derivative of $(x^2 + 4e^x) \cdot x^4$.
6. Find the derivative of $(x^2 + 4e^x) \cdot x^4$ in a different way.

Quotient Rule: $(\frac{f(x)}{g(x)})' = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$.

1. Find the derivative of $\frac{3e^x}{x^5}$.
2. Find the derivative of $\frac{x^7}{x^3}$ using the quotient rule.
3. Find the derivative of $\frac{x^7}{x^3}$ using first some algebra and then the power rule. Should your answer agree with the quotient rule result? Why? Does it agree?
4. Many students **incorrectly** believe that $(\frac{f(x)}{g(x)})' = \frac{f'(x)}{g'(x)}$. Give an argument why this cannot be true.
5. Find the derivative of $\frac{10x^7 - e^x}{x^3 + 75}$ using the quotient rule.

Chain Rule: $(f(g(x)))' = f'(g(x)) \cdot g'(x)$.

1. Find the derivative of e^{2x}. What is f? What is g?
2. Find the derivative of e^{x+2x^2}. What is f? What is g?
3. Find the derivative of $(x^2 - 16)^{45}$. What is f? What is g?
4. Find the derivative of $(x^2 - 2)^2$. What is f? What is g?
5. Find the derivative of $(x^2 - 2)^2$ without using the chain rule. (Hint: use some algebra).
6. Many students **incorrectly** believe that $(f(g(x)))' = f'(g(x))$. Give an argument why this cannot be true.

General Exponential Functions Rule: $(a^x)' = \ln(a)a^x$.

1. Find the derivative of 2^x.
2. Use the logarithm rules to show that $a^x = e^{x \ln(a)}$.
3. Use $a^x = e^{x \ln(a)}$ and the chain rule to prove the general exponential functions rule.
4. Find the derivative of 15^x.
5. Find the derivative of e^x using the general exponential functions rule. Does it agree with our prior derivative of e^x?
6. Find the derivative of $3^x + x^3$.
7. Find the derivative of $\frac{3^x + 6x^4}{x+6}$.
8. Find the derivative of $6^{(x^7+18x)} + 50$.

Trig functions: $\sin'(x) = \cos(x)$, $\cos'(x) = -\sin(x)$.

1. Draw the graph of $\sin(x)$ and draw its derivative graph to argue why the derivative is equal to $\cos(x)$.
2. Draw the graph of $\cos(x)$ and draw its derivative graph to argue why the derivative is equal to $-\sin(x)$.
3. Find the derivative of $3\sin(x) + 6\cos(x)$.
4. Find the derivative of $\sin(2x - x^2) + 6$.
5. Find the derivative of $e^{\cos(x)}$.
6. Find the derivative of $\frac{1}{6\cos(2x)}$.

Logarithm with base e: $\ln'(x) = \frac{1}{x}$.

1. Draw the graph of $\ln(x)$ and draw its derivative graph to argue why the derivative is equal to $\frac{1}{x}$.
2. Find the derivative of $\ln(x^2)$.
3. Find the derivative of $3\ln(x^2 + x)$.
4. Find the derivative of $\sin(\ln(x))$.
5. Find the derivative of $\frac{5x}{\ln(x)+9}$.

General Logarithm: $(\log_a(x))' = \frac{1}{\ln(a)x}$.

1. Use the equation $\log_a(x) = \frac{\ln(x)}{\ln(a)}$ (why is this true?) to prove the general logarithm rule $(\log_a(x))' = \frac{1}{\ln(a)x}$.
2. Find the derivative of $\log_5(x)$.
3. Find the derivative of $\log_{10}(45x)$.
4. Find the derivative of $\frac{x\sin(x)+5^x}{6\log_5(4x)}$.