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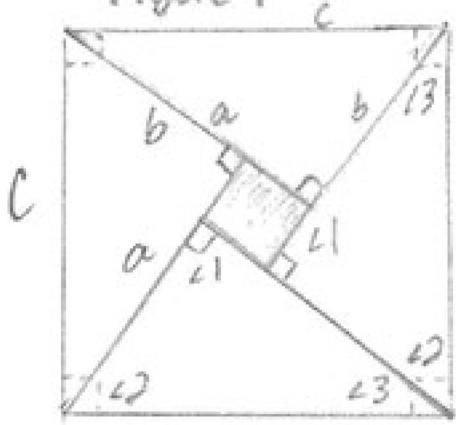
Pythagorean Theorem

The Pythagorean Theorem states that in a right triangle with sides a,b and hypotenuse c, $a^2 + b^2 = c^2$.

Proof: Start with four copies of the same right triangle. Each triangle is going to have an area of ab/2. Put them together to form a square with side c as shown in the first figure below. Notice, this square has a square hole in the middle with the side (a-b). In order to find out what c^2 equals, you have to take the sum of the area of the square hole $(a-b)^2$ and the area of the four triangles (4ab/2;2ab). You should get a^2+b^2 as the answer. You could also rearrange the four triangles and the little square to form two squares as shown in the second figure below. Since you are using the same shapes, the area is going to be the same no matter how you find it, and it's still $c^2=a^2+b^2$.

So you really have two proofs here, right? :)

Figure 1



▨ : (a-b)

$\angle 1 + \angle 2 + \angle 3 = 180°$

$\angle 2 + \angle 3 = 90°$

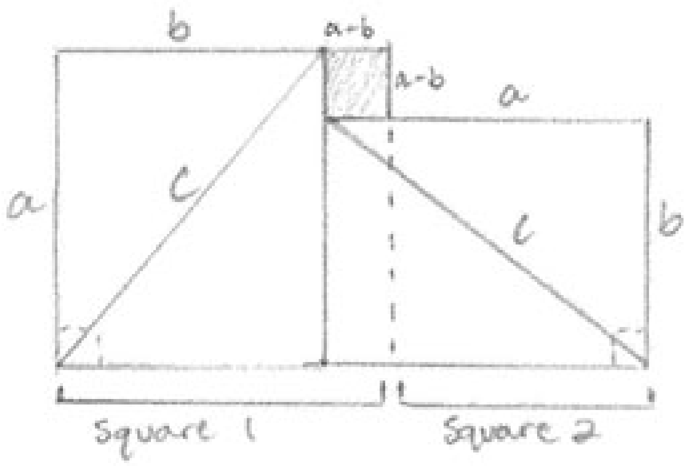
$\angle 1 = 90°$ $\angle 2 = 50°$ $\angle 3 = 40°$

$c^2 = (a-b)^2 + 2ab$

$c^2 = a^2 - 2ab + b^2 + 2ab$

$c^2 = a^2 + b^2$

Figure 2



We know both figures have the correct properties because they use the same shapes, four right triangles and a little square, to make the bigger square/squares. Each angle of a square is 90°. All of the angles of a right triangle equal to 180°. When you have a right triangle, you already know that one angle is 90° and that the other two angles, no matter what they are, have to equal the other 90°. Since all four triangles are the same in this proof, all of their angles have to be the same. When you put these right triangles together to form a square, one of the angles that is not 90° ends up perpendicular to the other angle that's not 90°. These angles are then added together to equal a 90° angle of a square. For instance, if you look at figure one, angle 2 of one triangle is perpendicular to angle 3 of another triangle. When these degrees are put together, they equal 90°.