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CA 1.6: Distinguishing Series and Integrals (Pg. 2)

2) To find the Sigma notation for each of these Series, I first noticed that the third one ($n=10$) is the same as the one on the reverse side, which we found to be:

$$\sum_{k=1}^{10} \left(\frac{1}{k}\right)^2$$

Then I noticed that the second ($n=4$) and forth ($n=20$) also dealt with squares, so I tried $n=4$ first. I saw right away that each term is multiplied by a constant $\frac{10}{4}$, so that will always be there in the sigma notation. Then I saw the first term was $\frac{1}{\left(\frac{10}{4}\right)^2}$, and the third was $\frac{1}{\left(\frac{30}{4}\right)^2}$, so I guessed the pattern would be: $\frac{1}{\left(\frac{10k}{4}\right)^2}$, and then I checked it with the other terms and it worked, so the sigma notation is:

$$\sum_{k=1}^{4} \left(\frac{10}{4}\right)\left(\frac{1}{\left(\frac{10k}{4}\right)^2}\right)$$

For $n=20$ I noticed the same pattern, only the constant is $\frac{1}{2}$, and the first and third steps are $\frac{1}{\left(\frac{1}{2}\right)^2}$ and $\frac{1}{\left(\frac{3}{2}\right)^2}$ respectively. That means this sigma notation is:

$$\sum_{k=1}^{20} \left(\frac{1}{2}\right)\left(\frac{1}{\left(\frac{k}{2}\right)^2}\right)$$

$n=2$ was a little trickier, after staring at it for a while I tried using $\frac{5}{5k^2}$, because the numerator is constant. This checked out with both terms, and the sigma notation looks like:

$$\sum_{k=1}^{2} \frac{5}{5k^2}$$ should be $(5k)^2$, right?

Now, to find a general expression I realised all of them were 1 over something squared (the 5 in $n=2$ can just be pulled out in front to give the $\frac{1}{5k^2}$). And 1 over something squared can be written as $\frac{1}{k^2}$. Now the hard part was finding a constant in common with all of them. Finally my eye turned to $n=4$, where the constant was $\frac{10}{4}$, so in that case the constant was $\frac{10}{n}$, which I then checked with all the others and it worked out, so the general sigma equation is;

$$\sum_{k=1}^{4} \left(\frac{10}{n}\right)\left(\frac{1}{k^2}\right)$$

yes, but instead of k you need to have $k \cdot \frac{10}{n}$ or you simplify, but then it should be $\frac{n}{10}$ in the beginning?

To determine if this was an integral or sum, I looked at the $\frac{10}{n}$, and thought that it looked a lot like the

$\frac{b-a}{n}$ term in the Riemann Sum of an integral. So in this case $b-a=10$, and the last two integrals have terms $b=10$ and $a=0$, so $b-a$ does infact equal 10 for those two. To figure out which one was right, since they are so similar, I wrote out both their Riemann Sums and simplified them to:

$$\int_0^{10} \frac{1}{x^2}\,dx \longrightarrow \sum_{k=1}^{n} \frac{n}{10k^2}$$

aha, here you have $\frac{4}{10}$, so was it just a typo before or is there a problem?

$$\int_0^{10} \frac{1}{10x^2}\,dx \longrightarrow \sum_{k=1}^{n} \frac{1}{10k^2}$$

I then checked the values of these sums with that of the ones above ($n=2$, $n=4$, $n=10$, and $n=20$) and found that the top integral worked for all of them, so the integral for the general n is:

$$\boxed{\int_0^{10} \frac{1}{x^2}\,dx}$$

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CA 1.7: Velocity and Distance Travelled (#1, 2, and 3)

1) To calculate the distance travelled of this equation I assumed we would use the integral, since we just spent all those packets working on it. However, I knew I could not blindly use it, so I just thought about why it would make sense to use it. This is what I came up with.

If this graph shows the miles per hour of the trip for three miles, the at $x=1$ it tells you you are going $f(x)$ miles an hour, which at 1 hour would be the total distance at that point (for example if you travel 60 miles/hour for an hour, you travel a total of 60 miles). Then to get the total distance you would just add the same thing for when $x=2$ and $x=3$. Fortunately, this process I just discribed is exactly what we have been doing with Riemann Sums and Integrals all along! This is how I was sure the integral would give us the total distance traveled.

also see if the below equation looks like a Riemann sum. In your case it doesn't look like we are

Also, the units of the equation $v(t)$ are in miles per hour, and the total distance travelled is just miles.

With all this in mind we calculated our Integral from $t=0$ to $t=3$ in the "app" and found the estimate to be 178.8 miles.

a way to improve this estimate would be to use more than 3 subdivisions of our Riemann Sum, so we would get a more accurate reading.

2) I wrote a mathematical representation of this as an Intergral and Riemann Sum:

urg!?

$$\boxed{\int_0^3 \frac{3}{n} f\left(\frac{3}{n}R\right) \Leftrightarrow \lim_{n\to\infty} \frac{3}{n}\left(\sum_{R=1}^{n} \left(-15\left(\frac{3}{n}R\right)^4\right) - \left(19\left(\frac{3}{n}R\right)^3\right) - \left(219.6\left(\frac{3}{n}R\right)^2\right) + \left(211.2\left(\frac{3}{n}R\right)\right)\right)}$$

should be $\int_0^3 v(t)\,dt$

3) Using the above limit of a sum, we found the exact distance traveled to be $\boxed{189 \text{ miles}}$, about 10 miles off of our original estimate.

I think you mixed the sum and the integral somehow?

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{b-a}{n} f\left(a + k\cdot\frac{b-a}{n}\right)$$

?