

Math 106 Calculus II
Course Activity I.7: Velocity and Distance Travelled

Purpose: To investigate how to calculate the distance travelled by an object if you know the velocity of that object.

Procedure: Work on the following activity with 1-2 other students during class (but be sure to complete your own copy) and finish the exploration outside of class.

1. Suppose on a recent car trip your velocity (in miles per hour) was given by the function $v(t) = -15t^4 + 96t^3 - 219.6t^2 + 211.2t$, where t is measured in hours. Your trip lasted from $t = 0$ to $t = 3$ hours. Estimate the total distance travelled by you on this trip. Use the following questions to help you with your estimate.
 - ✓ What simplifying assumption would help you make an approximation?
 - ✓ What are the units of $v(t)$? What are the units of distance travelled?
 - ✓ Can you improve on your current estimate? How?

2. Write a mathematical expression that represents the exact distance travelled on this trip.
3. Calculate the exact distance traveled using your answer to (2) and *Mathematica*. How close was your estimate?

In the following questions use *Mathematica* to calculate exact answers where necessary.

4. A woman goes for her daily walk on a straight trail – she always begins her walk at the beginning of the trail. Her velocity, $v(t)$ (in miles per hour), on a particular day is given by the following function:

$$v(t) = \begin{cases} 4, & 0 < t < 15 \\ -3, & 15 < t < 30 \end{cases}$$

where t is measured in minutes.

- a. What was the total distance that the woman walked on that day?
- b. How far was she from the beginning of the trail at the end of her walk?

5. A particle moving along a straight line has a velocity, $f(x)$ (in cm/sec), given by the following function:

$$f(x) = (-x^2 + 9x + 5) \cdot \sin x$$

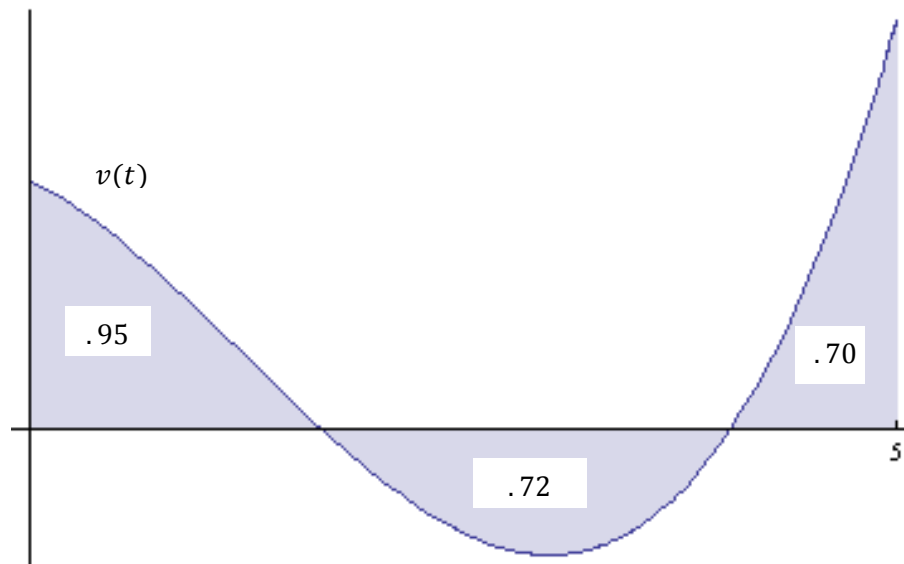
where x is measured in seconds.

- a. In total, how far does the particle travel between $x = 0$ and $x = 3$ seconds?

- b. In total, how far does the particle travel between $x = 0$ and $x = 5$ seconds?

- c. How far is the particle from where it started after $x = 5$ seconds?

6. The following graph shows the velocity, $v(t)$ (measured in miles per minute), of a car observed on a straight road from 0 to 5 minutes. The area between the curve and the x –axis in each segment is marked.



- a. After 5 minutes, how far was the car from its position at $t = 0$ minutes?
- b. Write a description of a scenario that would explain this velocity graph – extra points for the most entertaining story that is also correct!

Methods Practice: Complete the following question individually.

1. Suppose a boat's velocity (in miles per hour) was given by the function $v(t) = 5e^t(\sin(t) + 1)$, where t is measured in hours. The boat travels from $t = 0$ to $t = 5$ hours.
 - a. Estimate the total distance travelled by the boat on this trip. (This should truly be an estimate – do not use *Mathematica* to calculate an exact answer at this point in the question.) Use the following questions to help you with your estimate.
 - ✓ What simplifying assumption would help you make an approximation?
 - ✓ What are the units of $v(t)$? What are the units of distance travelled?
 - ✓ Can you improve on your current estimate? How?
 - b. Write a mathematical expression that represents the exact distance travelled on this trip.
 - c. Calculate the exact distance traveled using your answer to (2) and *Mathematica*. How close was your estimate?