Math 106 Calculus II

Course Activity I.6: Distinguishing Series and Definite Integrals

Purpose: To deepen and apply our understanding of what a definite integral is, and in particular, what differentiates it from a series.
Procedure: Work on the following activity with 1-2 other students during class (but be sure to complete your own copy) and finish the exploration outside of class.

1. Consider the following finite sums for increasing values of n:

- $n = 3$: $1 + \frac{1}{4} + \frac{1}{9} = \sum$ (write this sum in sigma notation)
- $n = 4$: $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} = \sum$ (write this sum in sigma notation)
- $n = 10$: $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots + \frac{1}{100} = \sum$ (write this sum in sigma notation)
- $n = 20$: $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots + \frac{1}{100} + \cdots + \frac{1}{400} = \sum$ (write in sigma notation)
- Write a formula in sigma notation for this sum for a general n: $\sum$
- The limiting value (as $n \to \infty$) of the sequence of finite sums above is (choose one of the following six options):

○ $\int_0^\infty \frac{1}{x^2}\,dx$ ○ $\sum_{k=1}^{\infty} \frac{1}{10k^2}$

○ $\int_0^{10} \frac{1}{x^2}\,dx$ ○ $\sum_{k=1}^{\infty} \frac{1}{k^2}$

○ $\int_0^{10} \frac{1}{10x^2}\,dx$ ○ $\lim_{n\to\infty} \sum_{k=1}^{10} \frac{1}{n^2}$

Developed at Rockhurst University, Mathematics Department, 2010. This course activity addresses material in Sections 5.2 and 9.2 of your book.

2. Consider the following finite sums for increasing values of n:

- $n = 2$: $\frac{5}{25} + \frac{5}{100} = \sum$ (write this sum in sigma notation)

- $n = 4$: $\frac{1}{\left(\frac{10}{4}\right)^2}\left(\frac{10}{4}\right) + \frac{1}{25}\left(\frac{10}{4}\right) + \frac{1}{\left(\frac{30}{4}\right)^2}\left(\frac{10}{4}\right) + \frac{1}{100}\left(\frac{10}{4}\right) = \sum$ (sigma notation)

- $n = 10$: $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots + \frac{1}{100} = \sum$ (sigma notation)

- $n = 20$: $\frac{1}{\left(\frac{1}{2}\right)^2}\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right) + \frac{1}{\left(\frac{3}{2}\right)^2}\left(\frac{1}{2}\right) + \frac{1}{4}\left(\frac{1}{2}\right) + \cdots + \frac{1}{25}\left(\frac{1}{2}\right) + \cdots + \frac{1}{100}\left(\frac{1}{2}\right) = \sum$

- Write a formula in sigma notation for this sum for a general n: $\sum$

- The limiting value (as $n \to \infty$) of the sequence of finite sums above is (choose one of the following six options):

  - ○ $\int_0^\infty \frac{1}{x^2}\,dx$
  - ○ $\sum_{k=1}^{\infty} \frac{1}{10k^2}$
  - ○ $\int_0^{10} \frac{1}{x^2}\,dx$
  - ○ $\sum_{k=1}^{\infty} \frac{1}{k^2}$
  - ○ $\int_0^{10} \frac{1}{10x^2}\,dx$
  - ○ $\lim_{n\to\infty} \sum_{k=1}^{10} \frac{1}{n^2}$