The tiling problem

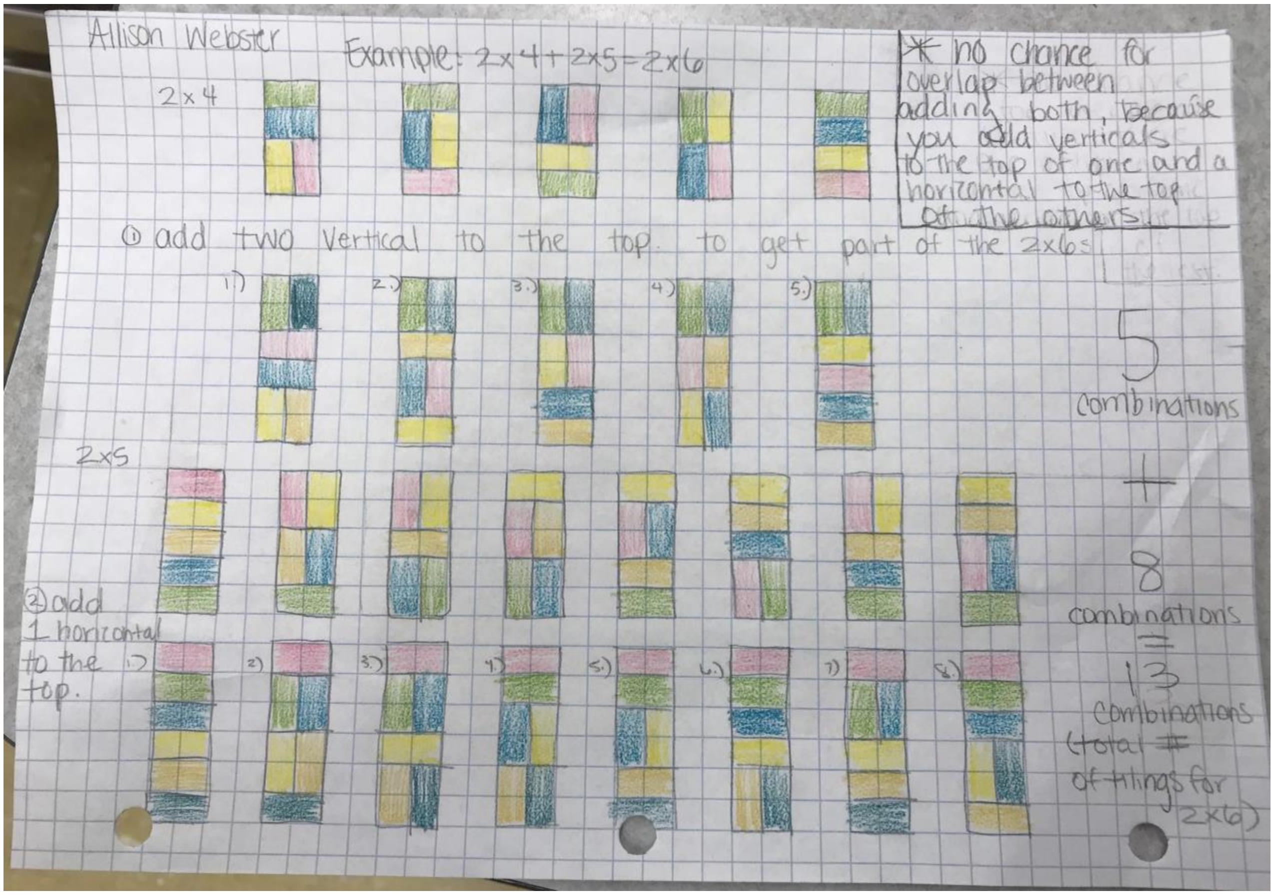
If you are given a 2xn rectangle you will be able to figure out how many ways you can tile it with a 2x1 rectangle with this equation:

$$2xn= 2x(n-1)+2\ x(n-2)$$

Meaning you need to add the past two numbers for the amount of combinations (ex. 2x1=1 combination + 2x2=2 combinations therefore a 2x3 would have 3 combinations because 1+2=3)

2xn	Number of combinations	The addition
2x1	1	
2x2	2	
2x3	3	1+2
2x4	5	2+3
2x5	8	3+5
2x6	13	5+8
2x7	21	8+13
2x8	34	13+21
2x9	55	21+34
2x10	89	34+55

You can also add the combinations, if you would like to figure out all of them. You have to set rules for you combinations.



Adding 1 horizontal to the 2x(n-1) then adding two vertical to the 2x(n-2) guarantees no overlap, because you have horizontals and verticals at the top which are not the same.

But to be sure you have all the combinations whether it is a 2x9 or a 2x81. As shown above we added the two previous total combinations for tiling to get the total number for the next tiling. Because this works constantly with lower numbers we can assume that it works with higher numbers.

We get all of the tilings for the largest one, because all of the possible combinations are put below the two vertical and the one horizontal. For the 2x(n-1) you know you have all the possible combinations then you just can add one horizontal to the top, which gives you all possible combinations with a horizontal tile on the top. Then for the 2x(n-2) you know all the possible combinations, the you just add two vertical to the top, which means you have all possible combinations once you add the 2x(n-1) and the 2x(n-2). Because, there is only two types of tiles you can put down, horizontal and vertical. With adding the one horizontal or two vertical to the top you know you have all possible combinations, because if you have two vertical at the top it is not the same as having a horizontal.