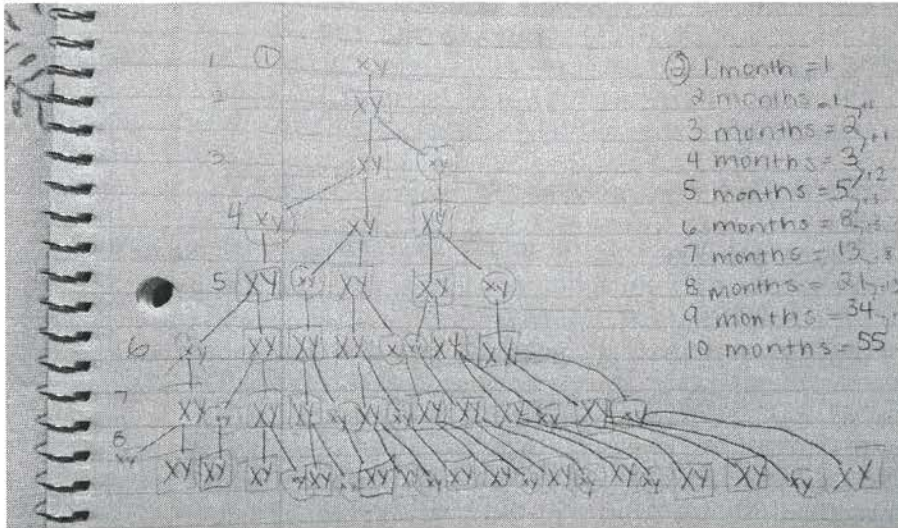


3
7
2
5
7

18
25

Chapter 1: Fibonacci Numbers Write Up

1.



2. The answer to Fibonacci's question of how many pairs will be produced in a year is 144 pairs. I figured this answer out by making a chart following the pattern I noticed forming in Investigation 1 which involved adding the two previous months total number of rabbit pairs together to get the total for the current month. I did this for 12 months and came up with the total number of pairs to be 144 pairs.

Months Number of Pairs

1	1
2	1
3	2
4	3
5	5
6	8
7	13
8	21
9	34
10	55
11	89
12	144

3. To determine the number of adult rabbits pairs for each of the months 2-8, I counted from the expanded Figure 1.2 that we drew in question one. I then made a table that showed the total of adult rabbits for each of the months 2-8.

Months Adult Rabbit Pairs

2	1
3	1
4	2
5	3
6	5
7	8
8	13

From this chart I was able to notice that for a given month (example month 5) you must simple add the two previous consecutive months total adult rabbit pairs together to get the total for that given month (month 5(3)=month 3(1)+ month 4(2)). This pattern works for all the months in the chart. I also noticed that all these numbers are Fibonacci numbers.

4. To determine the number of adult rabbit pairs in a give month, one must simply go back and look at the total number of rabbit pairs from the month before in the expanded Figure 1.2. By going back only one month in the breeding tree you see that only the juvenile rabbits are ageing and becoming adult rabbits. This leaves all the already current adult rabbits from that month to continuing being adults, which is what you are trying to determine for this problem.

5. To determine the number of juvenile rabbits pairs for each of the months 2-8, I counted from the expanded Figure 1.2 that we drew in question one. I then made a table that showed the total number of juvenile rabbits for each of the months 2-8.

Months Juvenile Rabbit Pairs

2	0
3	1
4	1
5	2
6	3
7	5
8	8

From this chart I was able to notice that this is the same pattern from the chart in question two, which shows the total number of adult rabbit pairs. The pattern is for any given month to find the total number of juvenile rabbit pairs you must add the two previous consecutive months totals of juvenile rabbit pair. This pattern works for the whole chart.

6. To find the total number of juvenile rabbit pairs for a given month, you must look back two months in the expanded Figure 1.2 and count the total number of rabbit pairs. This total will be your total number of juvenile rabbit pairs for the given month because it allows the adult offspring to reproduce every month. It also allows the juvenile rabbits to grow and start reproducing themselves.

7. By using Investigations 3-6 it is easy to explain why the number of rabbit pairs must follow the defining relation $F_n = f_{n-1} + f_{n-2}$ of the Fibonacci numbers. f_{n-1} is the equation form of looking back one month for the total amount of rabbit pairs in expanded Figure 1.2 as we discovered in Investigation 4. f_{n-2} is the equation form of looking back two month's for the total amount of rabbit pairs in expanded Figure 1.2 as we discovered in Investigation 6. By adding these two equations together you will get the next month's total, which is combination of adult rabbit pairs and juvenile rabbit pairs, giving you the total amount of rabbits for that month.

8. By continuing the pattern of Fibonacci numbers (adding the two previous consecutive months before) I was able to determine the 20th Fibonacci number to be $2584 + 4184 = 6765$. I have shown the first 20 Fibonacci numbers down below.

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4184, 6765

9. I believe it would be fairly difficult to determine the fiftieth Fibonacci number. I believe it would be difficult because it involves knowing all the previous 49 Fibonacci numbers. You would need to know the all of the previous 49 Fibonacci numbers because in order to get to the fiftieth Fibonacci number you need to add the 48th and 49th, and the get the 49th you need to add the 47th and the 48th and the get the 48th you need to add the 46th and 47th and so forth. This would be a very long process that would be very time consuming.

For questions 10-15, the pictures of pine cones and sunflowers are located on the last page of this packet.

10. By coloring every other spiral going clockwise in the pinecone drawing, I counted a total of 13 spirals.

11. By coloring every other spiral going counter-clockwise in the pinecone drawing, I counted a total of 8 spirals.

12. By coloring every other spiral going clockwise in the sunflower drawing, I counted a total of 34 spirals.

13. By coloring every other spiral going counter-clockwise in the sunflower drawing, I counted a total of 34 spirals.

+

Nice!

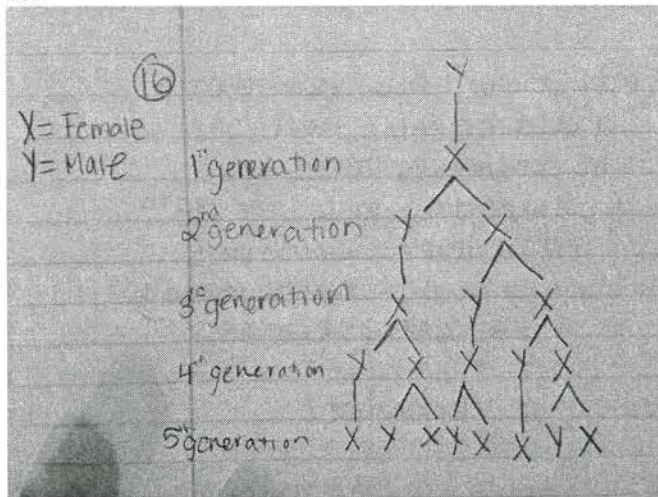
+

Reference
your pictures!!

14. A surprising fact about the total number of spirals found in Investigations 10-13 is that they are all Fibonacci numbers. Another surprising fact is that for the sunflower drawing the total numbers of spirals are the same in the clockwise and counter-clockwise direction. I believe this would have been the same for the pinecone drawings, but my lack of consistency between the drawings prevented that.

15. Some examples of Fibonacci numbers in nature include petals on a flower, leaf arrangements, and prickles on a pineapple plant.

16.

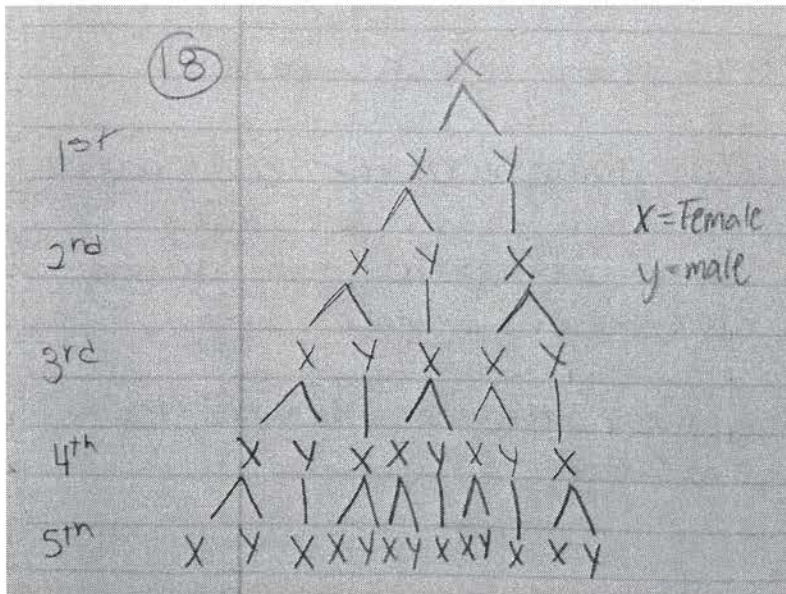


17. By looking at the family tree created in Investigation 16 and counting from there and knowing what you know about the bee reproduction pattern, you can determine that the male bee has:

- i) 1 parent
- ii) 2 grandparents
- iii) 3 great grandparents
- iv) 5 great- great- grandparents
- v) 8 great- great- great- grandparents

Looking at these numbers you can see that the lineage of a male bee follows the Fibonacci pattern.

18.



19. By looking at the family tree created in Investigation 18 and counting from there and knowing what you know about the bee reproduction pattern, you can determine that the female bee has:

- i) 2 parents
- ii) 3 grandparents
- iii) 5 great grandparents
- iv) 8 great- great grandparents
- v) 13 great- great- great grandparents

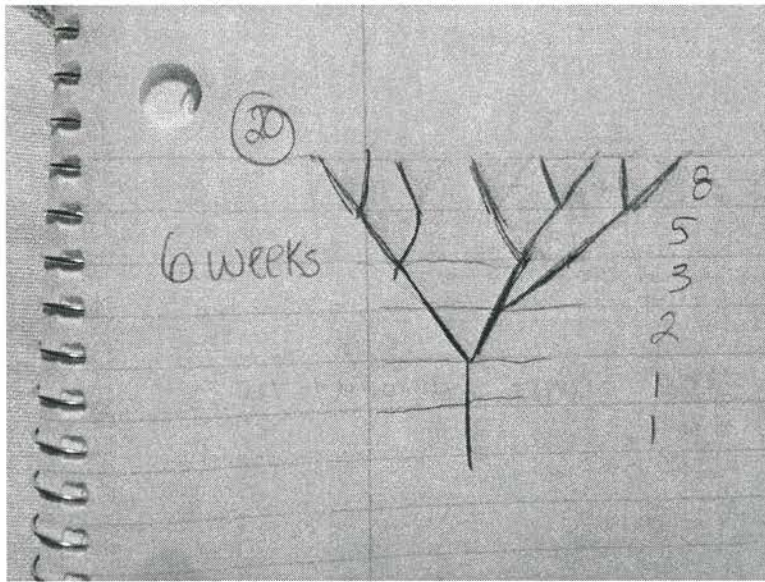
Looking at these numbers you can see that lineage of a female bee follows the Fibonacci pattern. This is also the same pattern the male bee follows as well.

Perfect.
Until ✓

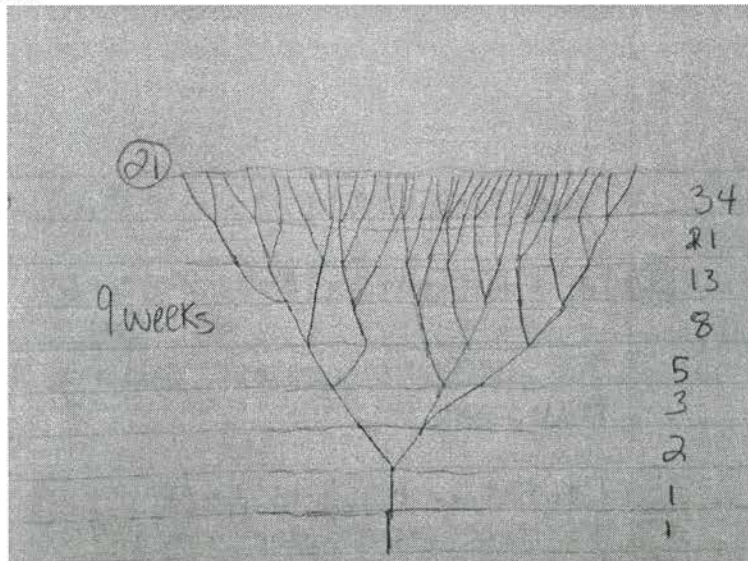
NO! It's off
by one generation!!

Oversharing.

20.



21.



22. By looking at the expanded Figure 1.4 that we drew in Investigations 20 and 21, I have noticed that the number of shoots from the Fibonacci plant after each week equals a Fibonacci number.

23. This problem is similar to that of the rabbit pair investigation we discovered in section 1.4. If you look back one week on the expanded drawing of Figure 1.4 you are able to count the total amount of fully grown shoots for that week. If you look back two weeks on the expanded drawing of Figure 1.4, you are able to count the total amount of still growing shoots. When you add these two totals together you will get the total amount of shoots for the current week. This is always a Fibonacci number because the amount of shoots from looking one week back is a Fibonacci

number and the amount of shoots from looking two weeks back is also a Fibonacci number. The pattern of Fibonacci numbers is that by adding two previous consecutive Fibonacci numbers together your answer will also equal a Fibonacci number.

24. By directly adding, the sum of the first three Fibonacci numbers is $1+1+2=4$.

25. By directly adding, the sum of the first four Fibonacci numbers is $1+1+2+3=7$.

26. By directly adding, the sum of the first five Fibonacci numbers is $1+1+2+3+5=12$.

27. By directly adding, the sum of the first six Fibonacci numbers is $1+1+2+3+5+8=20$.

28. By directly adding, the sum of the first seven Fibonacci numbers is $1+1+2+3+5+8+13=33$.

29. The sums in Investigation 24-28 are related to Fibonacci numbers because the differences between each set of consecutive Fibonacci numbers are also Fibonacci number and follow the Fibonacci pattern. Our conjecture regarding the value of the sum of the first n Fibonacci number is: $1+1+2+\dots+F_n = F_{n+2} - 1$.

30. $1+1+2+3+5+8+13+21=33+21$

By adding 21 to each side of our equation from Investigation 28, the new total is 54. It is easy to understand why adding 21 to each side will generate the correct result for the sum of the first eight Fibonacci numbers because 21 is the next Fibonacci number.

31. This is an equation that generalizes Investigation 30:

$$1+1+2+\dots+F_n + F_{n+1} = (F_{n+3} - 1)$$

32. This generalized equation proves our results in Investigation 29 because adding F_{n+1} is adding the next Fibonacci number. By adding the next Fibonacci your result will be still be $(F_{n+2} - 1)$ as we discovered in Investigation 29. This proves that no matter how long the sequence of Fibonacci numbers being added, the answer will always be one less than the next Fibonacci number.

33. This is a picture of Pascal's triangle with the next three rows from Figure 1.5.

How'd you get this?
This is #32. why? ✓

				1					
			1		1				
		1		2		1			
	1		3		3		1		
	1	4		6		4		1	
	1	5	10		10	5		1	
	1	6	15	20		15	6		1
	1	7	21	35	35	21	7		1
	1	8	28	56	70	56	28	8	1

34. This is how to expand $(x + y)^3$:

$$\begin{aligned}
 &(x + y)(x + y)(x + y) \\
 &(1x^2 + 2xy + 1y^2)(x + y) \\
 &(1x^3 + x^2y + 2x^2y + 2xy^2 + xy^2 + y^3) \\
 &1x^3 + 3x^2y + 3xy^2 + 1y^3
 \end{aligned}$$

This expanded version of $(x + y)^3$ shows that the coefficients are correctly given by the third row of Pascal's triangle because the coefficients in this equation are 1, 3, 3 and 1. In the third row of Pascal's triangle as seen in Investigation 33 the numbers are 1, 3, 3, and 1.

35. My conjecture about the expansion of $(x + y)^6$ is:

$$x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6$$

This is my conjecture because the coefficients (1, 6, 15, 20, 15, 6 and 1) match the 6th row of Pascal's triangle as seen in the picture from Investigation 33. I came up with this conjecture by following the pattern I saw forming in Investigation 34 and 35, which involved the exponents of x increasing when moving to the left and the exponents of y increasing when moving to the right.

36. These are the sums of each of the entries in the rows 1 through 6 of Pascal's triangle from Investigation 33:

$$\begin{aligned}
 1 &= 1 \\
 1 + 1 &= 2 \\
 1 + 2 + 1 &= 4 \\
 1 + 3 + 3 + 1 &= 8 \\
 1 + 4 + 6 + 4 + 1 &= 16 \\
 1 + 5 + 10 + 10 + 5 + 1 &= 32 \\
 1 + 6 + 15 + 20 + 15 + 6 + 1 &= 64
 \end{aligned}$$

After looking at these numbers, the pattern I noticed was the sum of each row is double to that previous to it.

Nice!

37. By directly adding the sums of the first shallow diagonal is $1+2=3$. By directly adding the sums of the second shallow diagonal is $1+3+1=5$.

38. The numbers that make up the third shallow diagonal are $1+4+3$ and their sum by directly adding is 8.

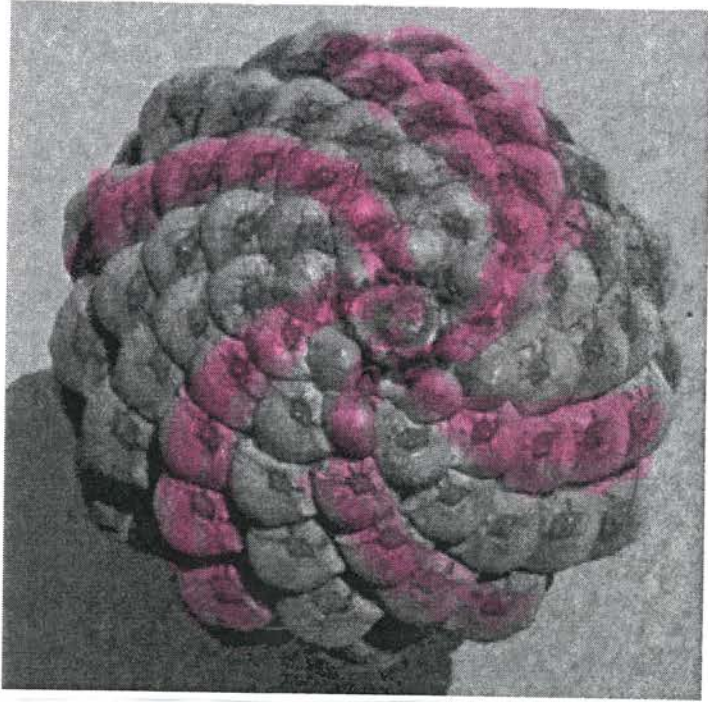
39. The numbers that make up the fourth shallow diagonal are $1+5+6+1$ and their sum by directly adding is 13.

40. The numbers that make up the fifth shallow diagonal are $1+6+10+4$ and their sum by directly adding is 21.

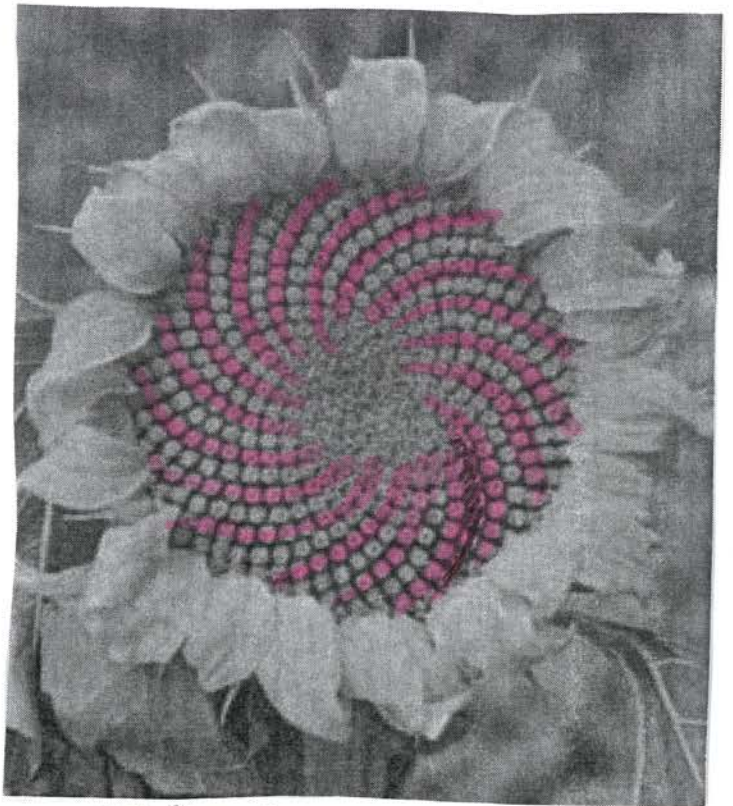
41. After looking at the sums of the first through fifth shallow diagonal I have noticed that the sums of each diagonal are Fibonacci numbers and they follow the correct sequence of the pattern.

in #37
- #70
✓

Clockwise

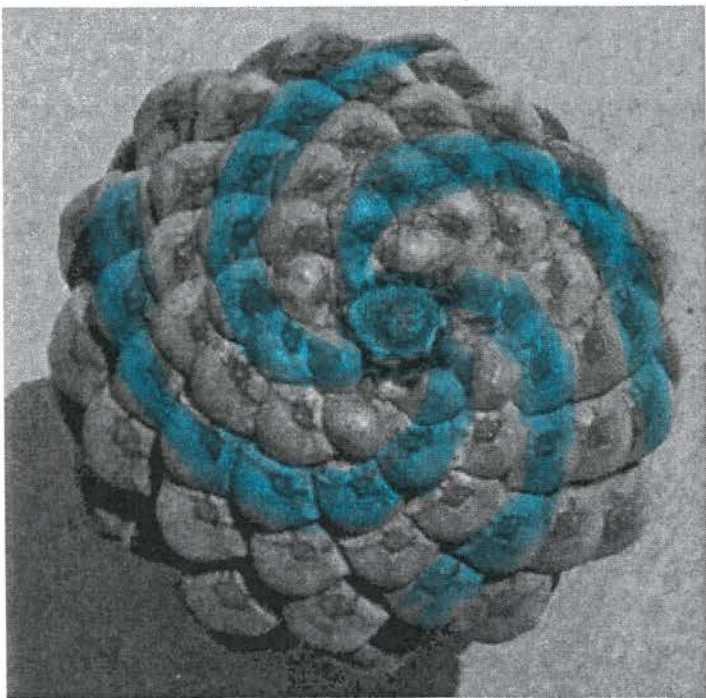


pinecone

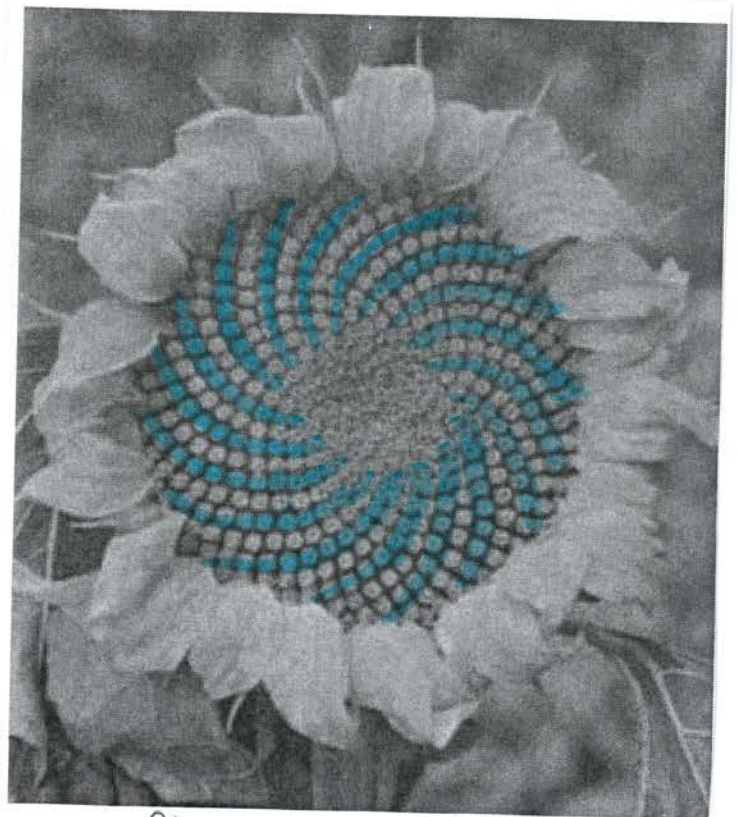


sunflower

Counter-clockwise



pine cone



sunflower

