Discovering the Art of Mathematics

Background

As reported by the Conference Board of the Mathematical Sciences (CBMS), over 182,000 two- and four-year college students were enrolled in Mathematics for Liberal Arts (MLA) courses in fall 2005. In four-year colleges the number of mathematics students in MLA courses is eclipsed only by calculus, college algebra, and introductory statistics. Moreover, MLA is one of the fastest growing college mathematics populations. While two-year college mathematics enrollments have increased by 33% over the period 1990 – 2005, two-year college MLA enrollments have increased 69%. More impressively, while four-year college mathematics enrollments have increased by 15% over the same period, four-year college MLA enrollments have increased over 66%. (CBMS, 2007)

The importance of introductory level college mathematics courses below the level of calculus has received significant attention. In their Crossroads in Mathematics: Standards for Introductory College Mathematics Before Calculus, the American Mathematical Association of Two-Year Colleges (AMATYC) undertook "the challenge of setting standards for curriculum and pedagogy in introductory college mathematics." [AMAYTC, 2005, from preface] In their 2004 Curriculum Guide, the Committee on the Undergraduate Program in Mathematics (CUPM) specifically highlights general education courses:

> General education [mathematics courses] are especially challenging to teach because they serve students with varying preparation and abilities who often come to the courses with a history of negative experiences with mathematics. Perhaps most critical is the fact that these courses affect life-long future workers and citizens. For all these reasons these courses should be viewed as an important part of the instructional program in the mathematical sciences. (CUPM, 2004, p. 27)

In addition to universal remarks about general education and introductory mathematics courses, it is important to make note of an earlier (1983) CUPM panel which specifically considered "mathematics appreciation courses… since these courses may play a major role in molding nonscientists' opinions of mathematics and its role in society." This panel was the last major public group which treated MLA independently in a detailed manner. It decided to "call attention to the importance of these courses and offer some suggestions on how they may be organized and taught effectively." (CUPM, 1983, p. 109) The panel notes, "Among all fundamental academic disciplines, mathematics is perhaps unique in the degree to which it is not understood (or is misunderstood) by students and even faculty from other areas of study." It goes on to say that the challenge "to place the discipline properly in the context of other human intellectual achievement… is enormous." (CUPM, 1983, p. 110)

"Mathematics appreciation courses" as they were called by the CUPM have more recently been subsumed by the title Mathematics for Liberal Arts. This genre of introductory, college-level mathematics courses also has come to include those where applications and/or quantitative literacy provide central content foci. There has been a dramatic, and healthy, increase in the volume of quality materials available which treat quantitative literacy and applications of mathematics. (COMAP, 2005; Tannenbaum, 2007)

Problems to be Addressed

We propose to address two significant problems we see confronting Mathematics for Liberal Arts – the first relates to content and the second pedagogy.

Mathematics for Liberal Arts Content

Before the promise of the CUPM recommendations on "Mathematics Appreciation Courses" were realized, the emergence of a quantitative literacy movement, an emphasis on applications of mathematics, and profound pedagogical changes made possible by the revolution in computer technology changed the landscape of Mathematics for Liberal Arts. In our view the majority of Mathematics for Liberal Arts (MLA) courses might more aptly be called Mathematics for Liberal Sciences.

We recognize and respect the importance of quantitative literacy, applications, and technology in all general introductory level mathematics courses. However, we feel that these topics have come to form a dominant paradigm for MLA. We do not believe this is a healthy balance for this genre.

Without real participation from across the disciplines in a mathematics across the curriculum movement, quantitative literacy and applications of mathematics are relegated to general education mathematics courses. This both entrenches the "two culture" divide and leaves Departments of Mathematics with the entire burden for these critical tasks. As these topics have become the focus in MLA, we lose contact with the natural, powerful bridge that has long kept mathematics as a liberal art and a cornerstone of higher education. This bridge is constructed from essential connections to the liberal arts; to philosophy, art and aesthetics, history, humanism (White, 1993), culture, and reasoning. As CUPM states,

> College students study the best paintings, the most glorious music, the most influential philosophy, and the greatest literature of all time. Mathematics departments can compete on that elevated playing field by offering and making accessible to all students intriguing and powerful mathematical ideas… Indeed, these courses [general education and introductory mathematics courses] should be developed and offered with the philosophy that the mathematical component of every student's education will contain some of the most profound and useful ideas that the student learns in college. [CUPM, 2004, p. 28]

Discovering the Art of Mathematics is designed to meet this challenge directly. The content which makes up the texts and supplemental materials in this library is significant, intellectually challenging mathematics which nurtures the natural connections to the liberal arts.

The most widely available, accessible writing focusing on connections of mathematics to the liberal arts, connections of mathematics to intellectual life, and on contemporary mathematics are the numerous trade books written for general, non-specialized audiences. While many are of the highest quality, they are generally stripped of "mathematical formulas and computations" to "make for easier reading." Any real understanding of mathematics cannot be stripped so bare of its essential elements.

Mathematics for Liberal Arts Pedagogy

Another drawback of the trade books for lay audiences just considered is that they are precisely long lectures transcribed into text. We subscribe to Paul Halmos' maxim "the only way to learn mathematics is to do mathematics"? Certainly policy documents have been reminding us we should. A "Basic Principle" cited in Crossroads of Mathematics is, "*Mathematics must be taught as a laboratory discipline.* Effective mathematics instruction should involve active student participation." (AMAYTC, 1995) Similarly, the Boyer Report states, "The inquiry-based learning urged in this report requires a profound change in the way undergraduate teaching is structured... Traditional lecturing should not be the dominant mode of instruction... The ideal embodied in this report would turn the prevailing undergraduate culture of receivers into a culture of inquirers, a culture in which faculty, graduate students, and undergraduates share an adventure of discovery." (Boyer Report, 1998, p. 16)

Generally, as teachers of MLA we have not done this. In 2005, 79% of Mathematics for Liberal Arts courses taught at two-year colleges were taught using the "standard lecture mode." (CBMS, 2007, p. 148) There is no evidence to suggest a different situation in four-year colleges where over 55% of Mathematics for Liberal Arts students are taught by faculty who are neither tenured nor in tenure-track positions. We remain, a quarter century later, largely in the situation CUPM reported on in 1983:

> Such courses are taken by a large number of students, frequently as their last formal contact with mathematics. Yet in most institutions they are given very low priority; they are frequently taught perfunctorily, without a clear set of objectives, by faculty who lack appropriate interest or credentials. (CUPM, 1983, p. 109)

Pedagogy is largely driven by the availability of appropriate curricular materials, particularly in settings like those just described. The explosion of technology in mathematics teaching has received significant support. As noted above, there is a significant body of high-quality materials available for quantitative literacy and applications of mathematics. In contrast, there is a much smaller selection of MLA texts, and their typical pedagogical approach make them quite hard to adapt to more inquiry-based and active learning approaches. This is a significant problem as an MLA course "requires more planning and preparation than almost any other mathematics course if it is to be successful." (CUPM, 1983, p. 111)

In Crossroads of Mathematics we are told, "development of new materials based on the standard set forth in this document is essential to lasting reform." (AMAYTC, 1995) In this project we have begun to do this. Our proposal outlines how we can continue our powerful progress. To begin, we provide a glimpse into the type of mathematical experience our materials provide for MLA students.

Vignette – A Day in Our Mathematics for Liberal Arts Classroom

It's still five minutes before class starts when I walk into the room. Each of the nine tables in the classroom has 2 or 3 students seated at it. Of the two dozen students already in the classroom a few are just sitting idly or chatting while getting organized, but most are already doing mathematics. They're already "getting their math on" as they often say. Their notebooks are open, already recording ideas, diagrams, data, guesses, patterns, computations, and rationales. Soon these things will be transformed into more formal responses to investigative prompts, conjectures, proofs, and mini-essays.

Individually I greet the few students who notice I have arrived, put the quote of the day on the board, and tour the room to see how everybody's work is progressing. As I do, the room fills up to its 35 student capacity, and by the time class is supposed to "start" every student is doing mathematics. Typically it takes only a few classes at the beginning of the semester for the students to be drawn in to being the center of the mathematical experience, and this group is no different. I've said nothing to the class as a whole, given no direction for them to start working, yet each student/group has picked up their mathematical exploration just where they previously left off.

Our focus this semester has been patterns. Our most recent foray is into a more strictly mathematical landscape than usual – the prime numbers. We're looking for strings of primes generated by a cylindrical "quadratic number sieve" that each student has constructed out of paper expressly for this purpose.

I approach a group who is a bit behind and now all of its members are focused on our materials. As I expect, they're having trouble deciphering a suite of investigative prompts:

> So what else can we say about the distribution of the primes? Consider the sequence of numbers $10^6!+2$, $10^6!+3$, ..., $10^6!+10^6$.
>
> 46) Explain why the numbers in this sequence are consecutive numbers.
> 47) How many numbers are there in this sequence?
> 48) Show that each of the numbers in this sequence in composite.
> 49) Contrast your answer in 48) with the conclusion of our earlier investigation we called the twin prime conjecture.

I urge them to be a bit more active and write things out. Almost immediately one student proclaims, "I get it." She quickly takes on the role of teacher and the group is happily back on their own.

I approach another group who also has communication needs. "Our group was in the library last night and we were having trouble figuring out how to explain this," one student shares. They are working on the investigations given in the box below. Sieves in hand, they show me exactly what they had discovered. As they articulate their ideas verbally, it becomes clear how they can construct a clear, coherent, well-supported descriptions and rationales for the results of their investigation – with very little guidance from me.

66) Show that the first differences of the values of the quadratic $q(n) = n^2 + n + c$ will be the same as those of the function $f(n) = n^2 + n$ and $s(n) = n^2 + n + 17$ we investigated above regardless of the value of *c*.
67) Rotate the sieve so the sieve setting is 17. What do you notice about the entries in the windows? I.e. precisely how do these entries relate to results of earlier investigations?
68) Repeat 67) with a sieve setting of 0.
69) You should see a critical relationship between entries in the windows for a given sieve setting and the functions *q* parameterized by the value of *c*. Describe this relationship precisely.

I continue around the room to a group who I notice is less active than normal and query them about their progress. "We don't know what to do, how to get conjectures for this goal." They point to the prompt:

Goal: Understand the ways in which functions $q(n) = n^2 + n + c$ generate primes as outputs depending on the value of the parameter *c*.

I ask them to show me what they've done. They use their sieves to illustrated that they clearly have several well-developed strategies for looking for strings of primes. "Why aren't you using even settings?" I ask. "Because all the results are even. They're not prime." "Shouldn't that be a conjecture? Isn't that part of your understanding of the goal?" I can see the change wash over them as they begin to talk excitedly about other parts of their strategy that quickly become conjectures that are recorded in their notebooks.

As I pass another group one of the students tells me, "Yesterday in my criminal justice class the professor was talking about burden of proof in civil versus criminal trials. That's kind of like all that stuff we've done about inductive and deductive reasoning, right?" The group and I talk about the parallels. As we talk I realize a contentious debate about something mathematical is brewing in a group across the room. I'm not close enough to overhear the details, but I know this is fundamental to real learning. I'll check on them in a bit to see what they have concluded.

Class goes on happily like this for about thirty minutes until I call for a pause so I can make a few announcements, tie some mathematical issues together and talk about how we'll evaluate our current body of work. Since we've done a "notebook quiz" and an "oral exam" for the last two sections respectively, the students will write up all their results for summative assessment of this section. I show them the lead article "Prime Number Patterns" (Granville, 2008) in the April *American Mathematical Monthly* and the cover of the May *Notices of the American Mathematical Society*. I'm excited because the former describes tantalizing progress towards solutions of questions we're investigating and the latter shows a two-dimensional Sieve of Eratosthenes that bears striking resemblance to the quadratic number sieve we've built and used for the past few weeks. The students see that the course materials have already given them a context for and connection to these new papers. I'm eager to see their essay responses to summary questions that close this section that will be handed in shortly:

Essay 1) How does it feel to be working on mathematics fundamentally related to two 1$ Million Millennium Prize Problems?

Essay 2) In your opinion, how do you think mathematicians feel about the status of our understanding of the distribution of the primes?

Essay 3) You've read about the importance of primes in data encryption and security. Should this have some impact on mathematicians efforts to solve the Riemann Hypothesis?

Essay 4) In this section you worked on mathematics that has captivated mathematicians through the ages. You have learned about some of the history and contemporary progress. Compare and contrast mathematicians' progress in this area with the progress of practitioners in major area of focus in their different field of thought.

The first comment when we get back to "getting our math on" gives me every confidence their responses will be good because one student calls out, "Hey Doc." As I approach she asks, "Is that Wiles guy who solved Fermat's Last Theorem working on this stuff we're doing too? This is number theory too. He didn't get $1 Million for his problem, I'd be working on this Riemann Hypothesis if I were him." The student smiles and her group members nod in agreement.

Proposal: An Inquiry-Based Mathematics for Liberal Arts Curriculum

The vignette above is not hypothetical. It is what we are currently doing. The featured course is Westfield State College's mathematics for liberal arts (MLA) course, MA110 – Mathematical Explorations. The featured class is an "Accuplacer section" – a class that failed our College's placement exam! The opportunity to offer MLA students a mathematical experience like this is only possible because we have available a small body of appropriate curricular materials that have been developed during the initial stages of this project. Sadly, the extant materials provide only fleeting, extremely localized, and prohibitively time-intensive opportunities to offer such a learning environment in MLA.

We are responding with this **Phase I** proposal whose main project component is a focus on the **creation of learning materials and teaching strategies** for Mathematics for Liberal Arts students. This is a three-year proposal with the following goal:

Central Project Goal – Develop, test, refine, and distribute a library of ten inquiry-based learning guides and supplemental teacher resources entitled Discovering the Art of Mathematics.

Each of these guides will be texts of approximately 100 pages. They can be used largely independently of each other and any two would provide appropriate material for a typical semester-long MLA course.

The goal of the Discovering the Art of Mathematics library is to provide compelling, high-quality curriculum materials and supplemental teacher resources with the following characteristics:

Content Goal - Our library will provide compelling, high quality curriculum materials and teacher resources which are engaging, intellectually challenging, and nurture in-depth explorations of mathematical topics which demonstrate the continuing role of mathematics as a cornerstone of the liberal arts tradition. This liberal arts focus includes: the role of mathematics as an intellectual pursuit, its continuing impact in shaping history, culture, logic, philosophy, and knowledge, its status as humanistic and aesthetic discipline, and its extensive contemporary growth.

Pedagogy Goal – The pedagogy of our library is radically student-centered, providing a striking alternative to traditional texts which are generally structured around a lecture dominant mode of teaching. By pragmatically employing insights from many different inquiry-based and active learning traditions, our approach supports a continuum of individual teaching styles without compromising the student focus. We expect this approach to promote a broad range of meaningful, positive cognitive, meta-cognitive, and affective student gains.

Both of these goals have been strongly informed by deep, long-term commitments to teaching MLA at the college level. These experiences were the genesis for the initial phase of this project, which was supported by two sabbaticals and small local grants. This initial phase has resulted in the development, field testing, and revision of three manuscripts which will serve as the first three volumes in our Discovering the Art of Mathematics library:

Discovering the Art of the Number Theory, 107 pages, by Julian F. Fleron (Fleron, 2003),

Discovering the Art of Knot Theory, 89 pages, by Philip K. Hotchkiss (Hotchkiss, 2008), and, Discovering the Art of the Infinite, 50 pages, by Julian F. Fleron (Fleron, 1998).

These texts and their sequels form the foundation of our project. We encourage you to work through a few sections of these texts, either via the selected excerpts included in the appendix or the full texts which are available online as .pdfs via http://www.wsc.ma.edu/renesse/bookgrant/ .

Evaluation of the initial stage of this project, given in the "Justification" section below, has been quite positive. We believe the initial stage demonstrates that our approach is viable, sustainable, and successful. It has also provided the necessary experience and direction needed to undertake the much larger project phase proposed here. We now describe the major components of our project.

Content

MLA is generally a "terminal" mathematics course. As such we are not constrained by typical content and coverage demands. Therefore, we can, and have, focused solely on the two goals above as we have considered potential content as vehicles. Work on both the initial stage of this project and on this proposal has brought appropriate topics to the fore. These topics have coalesced into organizing themes for each of the ten volumes: the infinite; number theory; knot theory; patterns; music; reasoning, truth, logic, and certainty; geometry; calculus; puzzles and games; art and sculpture. These themes allow us to realize our content goal described above.

Outlines for each of the proposed volumes are included in the appendix. Initial bibliographies for each of the proposed volumes are included in the "Content Bibliography" which is part of the general proposal bibliography. Note that because this is an extensive, three-year project, individual volumes are at different stages of development and this will be reflected in the outlines and bibliographies. This and additional information about this library, including the three manuscripts already developed, are available at our project website http://www.wsc.ma.edu/renesse/bookgrant/ .

Pedagogy

Years of experience have shown us that for the MLA audience, guided discovery immediately restructures the classroom dynamic between teacher and learner. We utilize this approach as a pragmatic version of the Socratic dialogue adapted to large audiences. As articulated in our series' Preface entitled "Notes to the Explorer", each book is:

> Really a guide. It is a map. It is a route of trail markers along a path through the world of mathematics. This book provides you, our explorer, our heroine or hero, with a unique opportunity to explore this path – to take a surprising, exciting, and beautiful journey along a meandering path through a mathematical continent named Number Theory. [Or Patterns. Etc.]

The guided discovery component is only part of our pedagogical approach. We use it to replace classroom lectures and narrative texts. As illustrated by the vignette above and our already developed materials, students are also engaged in more open-ended investigations. As described in the "Justification" section below, our approach is informed strongly by research and practice from many different inquiry-based and active-learning movements. As is also described below, our materials are developed to support a continuum of pedagogical approaches.

Creative Process

Our work on this project will be guided by our previous experience in the initial phase. It will take on the following form:

- Background Research and Curriculum Development – Extensive background research on both content and pedagogy is required to determine appropriate topics, themes, and vehicles for student investigation. Much of this broader work has been done. More focused work proceeds by developing small modules that are classroom tested. These smaller modules are then integrated into larger themes for the individual texts.

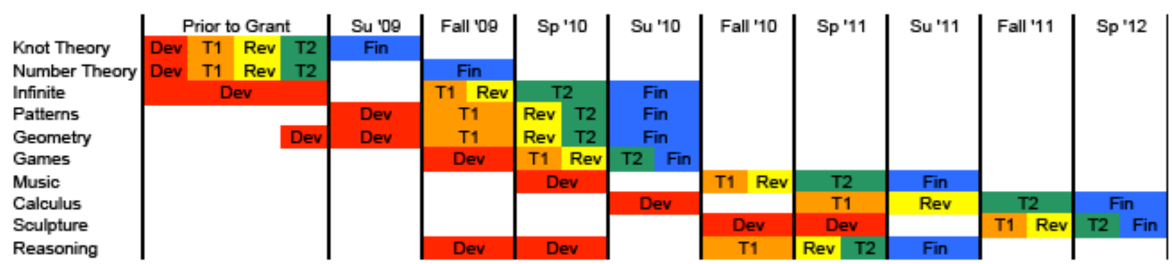
- Classroom testing by project staff and other faculty at Westfield State College will be ongoing through the development of the materials. This will include both formative and summative assessment of student gains as well as formative and summative student assessment of the materials. There will be significant classroom testing at the end of initial development as well as a second round of testing following major revisions.
- An external advisory board will provide feedback. In particular, each text will be both reviewed and beta-tested by at least one member of the advisory board.
- Small scale revisions to the materials will be ongoing and major revisions will be made following classroom testing, external review, and beta-testing.
- Final revisions will be made in preparation for publication.
- Creation of appropriate teachers' materials will be ongoing throughout, informed strongly by the advisory board.

Experience has shown that at Westfield State College, where the teaching load is 12 credits per semester, the approximate workload for the completion of each text from beginning to end is approximately:

Research and Development (Dev)	7 Credits	Second Classroom Testing (T2)	1 Credit
First Classroom Testing (T1)	2 Credits	Final Revisions (Fin)	2 Credits
Revision (Rev)	2 Credits		

This is a total of 14 Credits per book. Three books have already passed through many of the stages and a fourth has already begun research and development, providing an expected workload equivalent to 106 credit hours. In conjunction with support from our home institution (including two expected sabbaticals), this grant will support this project mainly through academic year course releases and summer stipends.

A precise timeline for the distribution of workload by book topic, semester, and project staff is given in the appendix. A summary graphic which provides the timeline is given below:



<u>Dissemination</u>

As described more explicitly in the "Goals" section below, we expect dissemination to be extensive.

Our project Internet pages will be routinely updated. We will regularly participate in conferences, workshops, and meetings, presenting, organizing, and sponsoring when possible. In particular, we hope to organize a Special Session at the Joint Meetings in 2012.

We expect the entire series to be published. We have a number of potential publishers in mind as described below. The project has already spawned significant scholarly work resulting in publications and we expect this to continue.

We believe that there are many opportunities that may follow Phase I of this project, including a follow-up Phase II proposal.

Rationale

In this section we show that our project is grounded in a vital mix of research on teaching and learning, content knowledge expertise, an experienced and qualified project team, and extensive classroom teaching practice. As such, this project is perfectly aligned with "the cyclic model of the relationship between knowledge production and improvement of practice" that is the basis of the CCLI program. (CCLI, NSF 08-546, p. 5).

Exceptional Qualifications and Opportunity

Westfield State College provides the perfect environment for the research, development, and field testing of the curriculum materials that will make up this library. Our College is a comprehensive "People's College" whose teaching tradition goes back to its founder Horace Mann. It has a significant Liberal Arts component and a common core that requires students to take two mathematics courses. Letters of support from the President and Academic Vice President (see appendix) illustrate the College's strong support.

The Department of Mathematics' mission is compatible with our project. Its mission states, in part,

> The Mathematics Department offers its students an energetic and dedicated faculty who find the world of mathematics exciting and challenging, and coursework that provides them with opportunities to experience the wonders of mathematics themselves... The Mathematics faculty is proud of their commitment to creative classroom instruction and active learning... Many of these pedagogies, by requiring that students work together, help to create an inclusive, interactive learning environment where students develop important communication and interpersonal relationship skills.

Our Department offers a dozen courses that help fulfill the common core requirements. These include two mathematics for liberal arts courses: MA110 – Mathematical Explorations and MA111 – Mathematical Applications. Our Department offers as many as 12 sections of each of these courses per academic year. Hence, we have ample access to appropriate students and classes for development, testing, and evaluation. Our Department Chair is a strong supporter of this project – his letter of support is included in the appendix.

The Senior Faculty who make up our team are well-positioned to meet the pedagogical, mathematical, teaching, writing, and dissemination challenges of this project. We have broad and deep expertise in mathematics. We have significant background in pedagogy, mathematics education, and the history and philosophy of mathematics. We have significant experience teaching diverse audiences and we have the passion, intellectual interest, and commitment to make the goals of this project a reality. Detailed vitae are available on our project website: http://www.wsc.ma.edu/renesse/bookgrant/ .

Liberal Arts Focus

In their Crossroads in Mathematics, AMATYC tells us, "It is particularly important that liberal arts students understand the impact that mathematics has had on art, history, literature, and many areas of human endeavor." (Ch. 3, p. 22) CUPM offers similar guidance, "Students must come to understand the historical and contemporary role of mathematics, and to place the discipline properly in the context of other human intellectual achievement." (CUPM, 1983, p. 110)

Unfortunately, topics such as these, which are central to the liberal arts, "are ignored by many authors of mathematics for liberal arts texts. Although mathematics is central to modern science and modern western culture, most texts bypass these important aspects of mathematics." (White, 1998, p. 675)[1] Our project is compatible with vision of the CCLI program to provide "excellent science, technology, engineering, and mathematics (STEM) education for *all undergraduate students*" (CCLI, NSF 08-546, p. 4; our emphasis). This need for real integration of liberal arts mathematics into the heart of general education is hardly new. Morris Kline's warning is as relevant now, if not more so, than 50 years ago:

> Almost everyone knows that mathematics serves the very practical purpose of dictating engineering design. Fewer people seem to be aware that mathematics carries the main burden of scientific reasoning and is the core of the major theories of physical science. It is even less

[1] Despite being 30 years old this statement still applies. And, not surprisingly, the reviewed books are still in print, albeit one having changed audiences to Mathematics for Elementary Teachers.

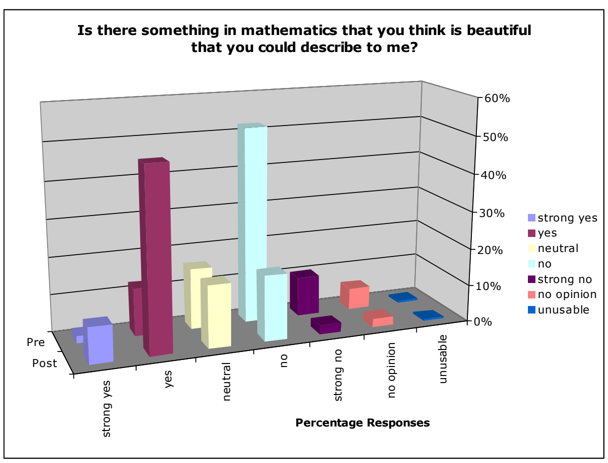
> widely known that mathematics has determined the direction and content of philosophical thought, has destroyed and rebuilt religious doctrine, has supplied substance to economics and political theories, has fashioned major painting, musical, architectural, and literary styles, has fathered our logic, and has furnished the best answers we have to fundamental questions about the nature of man and his universe... Finally, as an incomparably fine human achievement mathematics offers satisfactions and aesthetic values at least equal to those offered by any other branch of our culture. Despite these by no means modest contributions to our life and thought, educated people almost universally reject mathematics as an intellectual interest. (Kline, 1953)

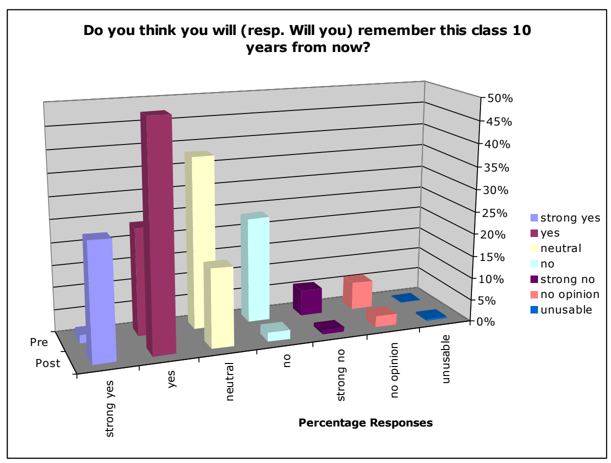
We will meet this challenge head on. Our content is chosen expressly to nurture the study of topics central to the liberal arts. As is clearly evident from our completed manuscripts and both the outlines and bibliographies for the proposed texts, our materials are infused with art, history, culture, humanism, philosophy, and literature throughout. For example, our texts will contain extensive student investigation of a number of revolutions in intellectual achievement: non-Euclidean geometry; higher dimensions and fractal dimensions; the understanding of the infinite and infinitesimals; the continuous versus the discrete; and fundamental limits to knowledge.

Our experience shows that our choice of content and pedagogy helps students "develop the view that mathematics is a growing discipline, interrelated with human culture, and understand its connections to other disciplines." (AMATYC, Intellectual Development Standard I-4.) The success of our approach compatible with CUPM, 1983, which reminds us:

> Liberal arts education, for a large percentage of the college educated population, is a rigorous, disciplined encounter with the best elements of man's history and culture. The major clientele of the mathematics appreciation courses are liberal arts students, and it is from their ranks that many of society's leaders will emerge. (CUPM, 1983, p. 110)

We have collected data from over 500 MLA students from 17 sections over 10 years in evaluating initial stages of this project. The complete data is included in the appendix. As illustrated by the data shown below, we are making significant progress with some of the challenges described in this section.





<u>Pedagogy</u>

In the proposal section above we noted how our approach beginning with guided discovery fundamentally changed the classroom dynamic as recommended by the Boyer Commission. (Boyer Report, 1998, p. 16) In this section we describe how this approach builds on the existing teaching and learning knowledge base.

Throughout this discussion, it is critical to remember that our audience is Mathematics for Liberal Arts students. A Socratic approach to teaching is ancient, but the efficacy of any active-learning approach to teaching is heavily dependent on audience and content. We have taken a pragmatic approach. Because

"Mathematics is something that one does" (Moise, 1965), restructuring our classrooms proceeded from the central activity we envisioned students engaged in – solving problems. Guided discovery is a perfect vehicle to help begin this transformation. In this regard, our approach nicely described by Freudenthal:

> Common sense and guided reinvention of mathematics will again pave the road to the answer. It will not be a simple answer since guiding reinvention means striking a subtle balance between the freedom of inventing and the force of guiding, between allowing the learner to please himself and asking him to please the teacher. Moreover, the learner's free choice is already restricted by the 're' of 'reinvention'. The learner shall invent something that is new to him but well-known to the guide. In any case reinventing is at least reinventing *something*. (Freudentahl, p. 48)

This approach is also guided by the heuristics of Lakatos, 1976 and similar warnings of Koestler, 1966, who reminds us, "The traditional method of confronting the student not with the problem but with the finished solution means depriving him of all excitement, to shut off creative impulse, to reduce the adventure of mankind to a dusty heap of theorems."

Of course, the guiding cannot be too strict (Freudenthal, 1991, p. 45-8; Borasi, 1996, p. 21). We have found that as guided discovery helps build student interest, confidence, and ability levels throughout the course the role of guided discovery can be decreased while other inquiry approaches come to the fore. The supplemental teacher materials play an important role in this transition by providing the appropriate resources. Please see the "Vignette" above, the excerpts in the appendix, and the completed texts available online for more examples.

In this phase the work of our project has been influenced by many important active learning traditions. These include literature in active learning in general (NRC, 2000; Donovan, 2005; Postman, 1969; Bickman, 2003; Davidson, 1989; Dubinsky, 1997), the Moore Method (Moise, 1965; Whyburn, 1970; Jones, 1977; Parker, 1990; Mahavier, 1999), inquiry-based learning, in a stricter sense than we have used the general term above (Audet, 2005; Lee, 2004), and problem-based learning (Amadore, 2006; Delisle, 1997; Duch, 2001). Borrowing from these many traditions, Our pragmatic approach insures that students will meet AMATYC's intellectual goal of engaging "in rich experiences that encourage independent, nontrivial exploration in mathematics, develop and reinforce tenacity and confidence in their abilities to use mathematics, and inspire them to pursue the study of mathematics and related disciplines." (AMATYC, 1995)

<u>Curriculum Availability and Accessibility</u>

Curricular materials that support the Liberal Arts content we believe is important for this audience – as above, excluding quantitative reasoning and applications with no slight intended – are in limited supply. Our content vision is shared by the <u>Mathematics Across the Curriculum Program</u> at Dartmouth College which was funded with $4 Million NSF grant. Namely, as "cries for increased numeracy of the undergraduate population have fed the development of mandatory general education courses in 'quantitative literacy' throughout the country, Dartmouth has taken a different and more difficult road, with the creation of a broad swath of courses designed to attract rather than conscript students into mathematics. This alternate path stems from a lengthy discussion of the goals of a liberal arts education." (Wallace, 2000) While of great value, we feel the materials developed in this program may not provide the accessibility needed for typical faculty who teach this course and typical students who take this course. <u>The Heart of Mathematics</u> (Berger/Starbird, 2000) provides excellent content, but does not address the pedagogy needs described in "Problems to Address" above. (Fleron, 2000)

As noted in "Problems to be Addressed", the challenges in pedagogy are dramatic since texts for this audience are generally written following a traditional lecture approach style. Encouraging faculty to adapt to using a more inquiry-based approach is "a change not undertaken lightly. Giving up the safety and authority of the podium can be unsettling for faculty accustomed only to a traditional, teacher-centered lecture format." (Duch, p. 9)

We are aware of only three broadly available resources which correspond to our content and pedagogy visions for MLA: Jacobs, 1994, Farmer, 1996, and Farmer/Stanford, 1995. The topics in the first are uneven, in need of revision, and the level of the book is generally below an appropriate level for a college audience. The latter two books are excellent, but generally at a level that is too advanced for the typical MLA student. While one can piece together inquiry-based materials for an MLA course for a semester or two from these texts, it is a lot of work with uneven results that cannot be broadly utilized. In short, extant materials are not sufficient to support the content and pedagogy demands we have described.

Discovering the Art of Mathematics will provide the requisite texts and supplemental materials for faculty to broadly integrate inquiry-based approaches in MLA. The materials developed as part of this project support a continuum of pedagogical approaches to provide broad accessibility without compromising their student focus. For example, the texts together with the supplemental materials provide ample material for both teacher led discussion and homework/assessment. Hence, faculty and departments new to inquiry-based learning with this audience can gradually incorporate it into their teaching. On the other end of the continuum, one can use the approach described in the vignette which relies heavily on a strict inquiry-based. This flexibility will encourage broader adoption of our materials.

General Education Content

In their Crossroads in Mathematics, AMATYC provides seven "Standards for Intellectual Development". They include the following:

- I-1 Students will engage in substantial mathematical problem solving.
- I-3 Students will expand their mathematical reasoning skills as they develop convincing mathematical arguments.
- I-5 Students will acquire the ability to read, write, listen to, and speak mathematics.
- I-7 Students will engage in rich experiences that encourage independent, nontrivial exploration in mathematics, develop and reinforce tenacity and confidence in their abilities to use mathematics, and inspire them to pursue the study of mathematics and related disciplines. (AMAYTC, 1995)

As illustrated in our vignette, each of these plays a central role in our classroom. Our materials are developed to insure these standards play a similarly important role in classrooms that use these materials.

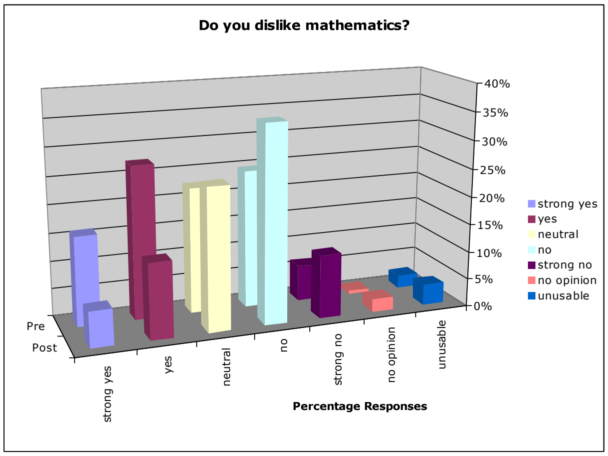
Contemporary Mathematics

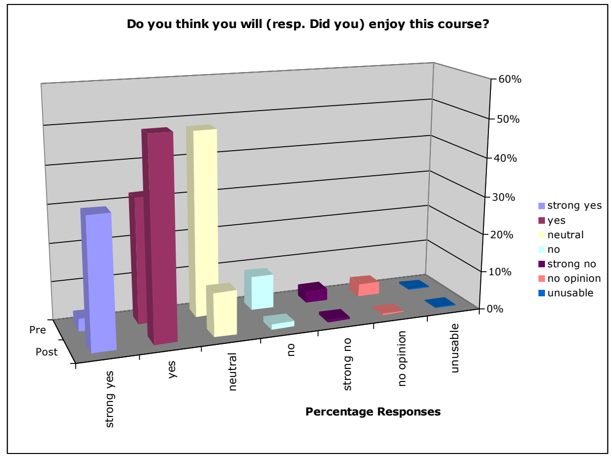
Crossroads in Mathematics recommends, "Whatever topics are selected should present fresh mathematics to students rather than a rehash of previously studied topics." (AMATYC, Ch. 3, p. 20) Similarly, MLA courses should stress: "The nature of contemporary mathematics, the recent emergence of several mathematical sciences, and the existence of a large body of interesting writing about mathematics." (CUPM, 1983, p. 111) As described in our goals below and illustrated by our outlines, bibliographies, and referenced work, the content of the library will largely be mathematical topics students have not previously encountered and will prominently include mathematics from each of the last three centuries.

Public Perception

In describing the broader impacts, the RFP asks, "What may be the benefits of the proposed activity to society?" (CCLI, NSF 08-546, p. 11) Public perception of mathematics in this country is not at all positive. Moving Beyond Myths warned us, "Harmful myths about mathematics metastasize to the body politic, spreading ignorance and excusing underachievement throughout society. Efforts to eradicate these pernicious myths will require sustained support at all educational levels, but especially in colleges and universities where society's leaders are educated." (CoMSY2000, 1991, p. 12)

As illustrated by the date shown below, our innovative approaches to content and pedagogy seem to be helping us make fairly dramatic affective gains with this audience. Complete data is contained in the appendix. As described in the section on goals and evaluation, we will collect additional data measuring affective gains as part of this project.





Goals and Evaluation

We are now in a position to describe how our general goals described in the "Proposal" section give rise to particular project goals and measurable outcomes that will be evaluated. Following each of the goals below we identify by number the specific tools (see "Project Evaluation: Assessment Tools" below) we will use for assessment.

There are a number of goals which relate to the broader impact of the project. These include:

- The creation of the first body of inquiry-based curriculum materials that provide a sufficient alternative to the pervading lecture-approach used in Mathematics for Liberal Arts (MLA) courses. This practical alternative will provide pedagogical flexibility so it can be broadly applied. (1, 4 - 8)
- The creation of innovative curricular materials for MLA that appropriately supports the central, long-standing and ongoing role of mathematics in the liberal arts tradition. (1, 3 - 7)
- The creation of meaningful, high-quality curriculum materials which successfully integrate contemporary mathematics into courses for non-specialists. (1 - 7)
- The creation of inquiry-based curriculum materials that are also appropriate for a large number of secondary audiences, including: mathematics for elementary education, independent study students, gifted and talented high school students, and home schooled students. (1, 3 - 5)
- Helping to build a larger community of teacher/scholars employing inquiry-based learning in a diversity of lower division college mathematics courses. (3 - 8)
- Helping to build a more active community of teacher/scholars who reclaim the tradition of the liberal arts as a balanced part of our mathematics curriculum, particularly in MLA courses. (3 - 8)
- Providing a baseline of appropriate inquiry-based curriculum materials for MLA which enable other faculty to contribute materials, spurring the creation of a larger, dynamic, library of materials. (6 - 8)
- The broad dissemination of the materials described above. (6 - 8)

The cornerstone of all of this work is certainly student learning. There are a variety of student learning goals which will be supported by the use of the Discovering the Art of Mathematics library. The most central student learning goals fall naturally into four main categories.

While there are local content goals for each of the volumes in the library, there are universal content goals as well, including:

- Students will understand the role of mathematics as an intellectual pursuit, its continuing impact in shaping history, culture, logic, philosophy, and knowledge, and its status as a humanistic and aesthetic discipline. (1, 2, 4)
- Students will understand the ubiquitous role of mathematics in the world around them. (1, 2, 4)
- Students will learn that mathematics is a vital, rapidly growing, field of intellectual inquiry with a dedicated cohort of practitioners. (1, 2, 4)

The many cognitive goals include:

- Students will strengthen their reasoning skills. (1, 2, 4)
- Students will strengthen skills in reading, writing, and speaking. (1, 2, 4)

Strengthening several metacognitive goals is also central to our approach. These include:

- Students becoming more self-monitoring, reflective learners. (1, 2, 4)
- Students becoming more effective problem solvers. (1, 2, 4)
- Students taking greater personal responsibility for learning. (1, 2, 4)

For this particular audience affective goals are particularly relevant. They include:

- Students will gain a healthy, balanced perception of mathematics. (1, 2, 4)
- Students will improve their mathematical confidence. (1, 2, 4)
- Students will approach mathematics more positively, aware of the negative impact of broadly held societal views. (1, 2, 4)
- Students will be capable of and interested in considering mathematics outside of the confines of the classroom, understanding the value of life-long learning in mathematics. (1, 2, 4)

Project Evaluation: Assessment Tools

The evaluation of this project will have many different components. They are numbered here to correspond to the numbering above.

1. Formative Assessment by Students during Research, Development, and Field Testing – As noted above, we have a perfect environment in which to develop and test these materials. As we have done in initial phases of this project, background research, development, and field testing of these materials will be done in close collaboration with students and faculty in these courses. In particular:

    a. Students will provide real-time formative feedback as they work through draft materials in class and outside of class.
    b. Students will be prompted with open-response questions which encourage them to reflect on varied content and learning experiences. Data generated from these reflective statements will be coded and used in the evaluation and appropriate revision of materials.

2. Formative and Summative Evaluation of Students – Significant student data has been collected via surveys, journaling, and many other forms of assessment over more than ten years. (The appendix includes the survey data.) We will continue and expand this work in the following ways:

    a. Under the guidance of our Evaluation Consultant we will revise our survey to help provide additional data to evaluate the affective goals above.
    b. Our Evaluation Consultant will also help us develop pre- and post-tests that will help provide reliable, psychometrically sound measures of content understanding, cognitive student gains, and metacognitive student gains.
    c. We will continue to require significant student journaling in addition to the writing in 1.b. These journals will be coded and used in the evaluation and appropriate revision of materials.
    d. We will continue to assess student learning through homework, quizzes, exams, projects, and posters. Each of these areas provides valuable formative assessment of the efficacy of our work.

    It should be noted that the scope and timeline of this project make large scale, controlled student evaluation unfeasible. At the conclusion of Phase I of this project, such evaluation will be the natural subject of a Phase II grant which includes "Assessing Student Achievement" as a component focus.

3. Advisory Board Review – Our advisory board will include several faculty who review materials and provide general guidance. Our advisory board will also include nationally recognized, grant-supported external reviewers. Each volume will be reviewed by at least one of these external reviewers.

4. Advisory Board Beta Testing – Beta testing will happen both locally with internal testers and globally under the direction of nationally recognized, grant-supported beta testers who serve on our advisory board. Each volume will be beta tested by at least one of these external reviewers in an appropriate MLA or cognate course. All beta test sites will report survey data, pre- and post-test results, and will provide detailed written evaluation of the beta testing.

5. Peer Review – We will continue to disseminate drafts and encourage voluntary review, beta testing, and evaluation by interested teaching/scholars. Notable reviews already include:

   a. George Andrews, President of the American Mathematical Society (AMS), called Fleron, 2003 "delightful", noting our project is "tackling… head on" the problem of "what can be done to provide mathematics courses of some substance for students in the liberal arts."
   b. Ken Ono, Presidential Early Career Award Recipient and 2009 AMS Invited Address Presenter, said, "Fleron's manuscripts… are a very good indication of his commitment to education."
   c. Margie Hale, author of the R.L. Moore style book Hale, 2003, says she "has no doubts that many students, previously bored with mathematics, will find the explorations in these texts engaging."

   See the appendix for the full text of these letters of support.

6. Presentations and Workshops – Any precise details of dissemination plans are dependent on the work-product resulting from the body of this grant. We expect to submit proposals which will be peer-reviewed to provide evaluation of the impact and intellectual merit of our work. Appropriate venues for presentations are expected to include regional and national meetings of the Mathematical Association of America (MAA), AMS, AMATYC, and Legacy of R.L. Moore. Additionally, appropriate venues for focused workshops are expected to include Project NExT, MAA PREP, Chautauqua Institute, and MAA Short Course at the Joint Meetings.

7. Publications – As above, it is too early to provide specific details. However, we expect the full library of ten texts, supplemental teachers materials, and scholarly papers resulting from this project to be nationally disseminated via high-profile, peer reviewed venues. *This will provide external, peer-reviewed evaluation of our work.* For example:

   a. Publication of the entire series would be appropriate through:
      i. The MAA's *Classroom Resource Materials Series* where the PI has served as an Editorial Board Member,
      ii. The AMS's *Mathematical World Series* which published Farmer, 1995 and Farmer, 1996.
      iii. Commercial publishers, several of whom have approached us about our work, or,
      iv. Internet publication through *Journal of Inquiry-Based Learning in Mathematics,* the MAA's *Mathematical Sciences Digital Library (MathDL)*, or similar venue.
   b. Publication of selected content and work-byproduct would be appropriate through many venues, and we have already demonstrated progress in this area:
      i. Fleron, 1999 was a direct result of work on Discovering the Art of the Infinite (Fleron, 1998).
      ii. Both Fleron/Ecke, 2008 and Fleron, 2008 are direct results of work on this project. The former is in final stages of preparation for submission to the National Council of Teachers of Mathematics *Mathematics Teacher* and the later is in the process of dissection into two separate papers for submission to the *Mathematics Teacher* and MAA *Math Horizons*.
      iii. *PRIMUS* has published a number of articles on Mathematics for Liberal Arts (Sterger, 1976; Briggs, 1993; Aboufadel, 1994; Garrison, 1995; Doty, 1995; Gura, 1996; Grant, 1999; Grzegorzyk, 2005) and an article on our pedagogical approach would seem appropriate.

8. Phase II Proposal – Given the extensive scope of the curriculum materials at the heart of this project, we are focusing on a single project component – Creating Learning Materials and Teaching Strategies. As should be clear from our goals and evaluation above, we expect that this project will develop into a larger, Phase II project that will focus on all five CCLI Project Components. Review of a proposal for this extended project would provide another level of evaluation.

Advisory Board

Initial stages of this project have already involved outreach to many faculty. We will continue to nurture broad collaboration with the guidance of an advisory board. This board will include external advisors who review and beta test each of the ten volumes in our library as described in the "Goals" section above. To illustrate the quality and diversity of the full board that will be assembled, we have already received the following commitments:

Reviewers

Underwood Dudley, Professor Emeritus, DePauw University. Ph.D. in Mathematics from the University of Michigan. Past Editor of both the College Mathematics Journal and the Pi Mu Epsilon journal. Past MAA Polya Lecturer. Author and Editor of a number of books including Numerology: Or, What Pythagoras Wrought and The Magic Numbers of the Professor.

David Farmer, Director of Programming, American Institute of Mathematics. Ph.D. in Mathematics from Oklahoma State University. Author of the AMS *Mathematical World* books Farmer, 1996 and Farmer/Stanford, 1995.

Melissa A. Shepard Loe, Associate Professor of Mathematics, University of St. Thomas. Ph.D. in Mathematics from University of Minnesota. Past Vice President of the Minnesota Council of Teachers of Mathematics.

Emily Puckett, Associate Professor, Sewanee: The University of the South. Ph.D. in Mathematics from Duke University. Project NExT Fellow.

Robert G. Stein, Professor of Mathematics, California State University San Barnardino. Ph.D. in Mathematics Education from the University of Texas, Austin. Former Program Officer at the National Science Foundation. Currently President of the American Section of the International Study Group on History and Pedagogy in Mathematics.

Beta-Testers

Doug Ensley, Professor of Mathematics, Shippensburg University. Ph.D. in Mathematics from Carnegie Mellon University. Project NExT Fellow. Author of Mathematical Reasoning and Proof with Puzzles, Patterns, and Games (with J.W. Crawley). Editor for the *Digital Classroom Resources* section of the Mathematical Sciences Digital Library.

Vince Ferlini, Professor of Mathematics, Keene State College. Ph.D. in Mathematics from University of Michigan. Former Mathematics Department Chair at Keene State College. Long time high school mathematics and science teacher in the United States (7 yrs.) as well as in both East and West Africa through the Peace Corps (5 yrs.).

Jeff Johannes, Associate Professor, State University of New York at Geneseo. Ph.D. in Mathematics from Indiana University. Project NExT Fellow. Organizer of "Creating and Teaching Interdisciplinary Courses" session at 2007 Joint Meetings.

Harriet Pollatsek, Professor of Mathematics, Mt. Holyoke College. Ph.D. in Mathematics from the University of Michigan. Chair of the CUPM Writing Team for the 2004 CUPM Curriculum Guide. Co-author of a number of books including: Laboratories in Mathematical Experimentation: A Bridge to Higher Mathematics, Calculus in Context, and Case Studies in Quantitative Reasoning.

Charles F. Rocca, jr., Assistant Professor of Mathematics, Western Connecticut State University. Ph.D. in Mathematics from State University of New York at Albany. Project NExT Fellow. Organizer of the WCSU funded ARITHMOS History of Mathematics Colloquia Series.

Evaluation Consultant

Janet Gebelt, Associate Professor of Psychology, Westfield State College. Ph.D. in Developmental/Social Psychology from Rutgers University. Statistical consultant and evaluation consultant for many programs from 1991 - present.