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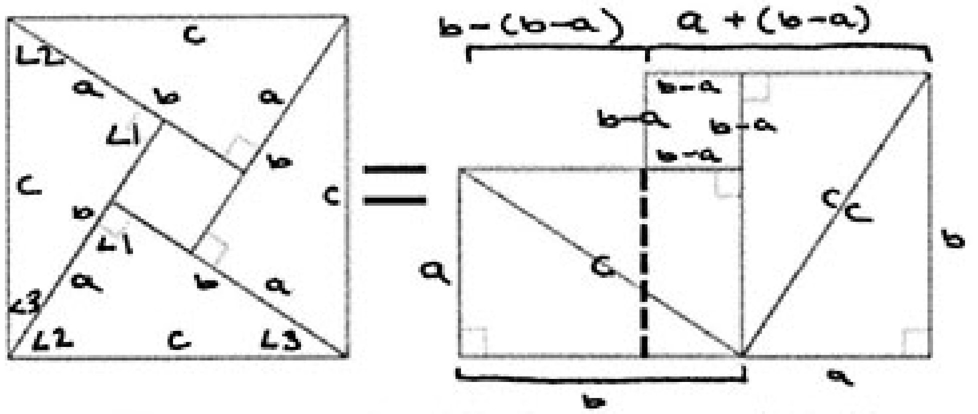
Class003
Prof. Fleron

Pythagorean Theorem

Conjecture: In a right triangle with sides a, b and a hypotenuse c, then $a^2+b^2 = c^2$.

Proof:



We cut a triangle while the paper was folded into quarters so we know all 4 of the triangles are exactly the same. The four triangles on the left, with an additional little square in the center, arrange into a square that represents c^2. We know this shape on the left is in fact a square because all the edges are equal and all the angles are 90 degrees. The edges are all c, which means they are equal. To prove all the angles are 90 degrees you have to know that a triangles angles add up to 180 degrees, you then subtract 90 degrees because <1 is a right angle. You are then left with <2 and <3, which add up to be 90 degrees. In the squares <2 and <3 touch each other so we know a 90 degree angle is formed, therefore the image on the left is definitely a square. We also know the little square piece in the middle is a square because the triangles legs are perpendicular to one another, so right angles are formed for every corner, and each edge is (b-a) so all the edges are equal as well.

The figure on the right represents the picture of c^2 rearranged into a^2+b^2, using exactly the same pieces. The dotted line represents where the figure is cut to create two individual squares. To the left of the dotted line is a^2 and to the right of the dotted line is b^2. The little square (b-a) is the key to creating the a^2 and b^2 squares. Originally in the figure on the right you have 2 rectangles with the dimensions a x b. By adding the little square that is (b-a) you are creating two squares, the square on the left with the dimensions a x (b- (b-a)) which comes out to be a x a, which is the square a^2. Then the square on the right becomes b x (a+(b-a)), which comes out to be b x b, which is your b^2 square.

Since the triangle legs are represented in letters you can put any value in for the letters, meaning this method would work for any right triangle, not just the one in this example. The two figures also represent how c^2 can be rearranged into a^2+b^2 using the same pieces to represent each side of the equation, proving the two sides (a^2+b^2 and c^2) are equal.