The following is excerpted from:

Inquiry-Based Learning and the Art of Mathematical Discourse

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By Christine von Renesse and Volker Ecke.

Discovering the Art of Mathematics (DAoM) is an NSF supported project that supports inquiry-based learning approaches for mathematics for liberal arts courses.

Free resources to support inquiry-based learning (IBL) in your classroom, both for IBL pedagogy and 11 learning guides for mathematics courses for liberal arts (MLA), are available online for free at

`http://www.artofmathematics.org/`

We offer individual **mentoring**, including a program where you can learn about IBL by visting us and our classes. For departments interested in IBL, DAoM offers **traveling professional development workshops**.

DAoM provides a wealth of resources for mathematics faculty to help transform their courses. Extensive online resources include classroom videos of IBL in action, sample student work, regular blogs about teaching using IBL and a regular newsletter. Opportunities for supported reviewing and beta testing are also available.

The DAoM curriculum consists of a library of 11 inquiry-based learning guides. Each volume is built around deep mathematical topics and provides materials which can be used as content for a semester-long, themed course. These materials replace the typical lecture dynamic by being built on inquiry-based investigations, tasks, experiments, constructions, data collection and discussions. Specific teacher notes and sample solutions provide facutly support.

Full information about the *Discovering the Art of Mathematics* project is available at

`http://www.artofmathematics.org`

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Use the following excerpt to learn about how to support students who are engaged in mathematical inquiry.

Talk Moves For Mathematical Conversations

Given the many different situations a teacher will encounter when interacting with students, what are the kinds of questions you can ask? We need a toolkit of questions to choose from.

Inspired by Suzanne Chapin's talk moves for whole class discussions, we decided to identify and organize talk moves for a conversation with just one student (or a small group). Our categories are chosen so that a teacher can easily decide which groups of questions to select from in a given situation. We recommend you pick one group at a time, find phrasings that feel natural to you, and try them out with your students.

1. Stages of Problem Solving: We will use Polya's stages of problem solving for the first few categories. We add the stages of "explaining" and "generalizing." For each category, we provide several possible questions (in italics) that you can use in your classroom.

1. Understanding the Problem: *Can you read the investigation out loud? What is this investigation asking you to do?*
2. Devising a Plan: *Can you predict what will happen? Can you guess and check? Can you draw me a picture? Is there a simpler version of this problem? Is there a pattern? What do you know about . . . ? What is the relationship of . . . to . . . ? What would happen if . . . ? What if the numbers were . . . ? Are any of your previous strategies helpful? When stuck, what has helped you before?*

3. Carrying out a Plan: *So if I use your strategy for . . . , what would happen? Is there another way to solve the problem so you can check your answer? Can you start a new page and organize all your information?*
4. Looking Back: *Is there another solution? How do you feel about your answer? How do you know your answer is reasonable?*
5. Explaining: *How can you convince me (us)? Why do you think that? Why did you ... ? Can you write an explanation for next year's students? What do you think would happen if . . . ? What is the difference between . . . and . . . ? How are . . . and . . . similar? What is a possible solution to the problem of . . . ? How does . . . affect . . . ? In your opinion, which is best, . . . or . . . —and why? Do you agree or disagree with this statement: . . . ? (Support your answer.)*
6. Generalizing: *Does . . . always work? Can you make a general claim or conjecture? What else would you like to know? What conclusions can you draw about . . . ?*

"Explaining" helps students find gaps in their reasoning and understand their own ideas and strategies in greater depth. Additionally, explaining a solution helps the teacher assess what a student actually understands. "Generalizing" is an important aspect of mathematical (and critical) thinking. It forces students to look at a bigger picture and think hard. Generalizing will also lead naturally to further mathematical questions that you might want to pursue in class.

2. General Moves to Clarify Thinking The following talk moves can be used at any time when a student is lost or stuck. They can also help the teacher understand what a student is doing and thinking. (You might learn something new here!) Two of Suzanne Chapin's general talk moves work well for a one-on-one conversation as well (revoicing & wait time), so we list them here.

1. Clarifying: *I am wondering about . . . Can you help me understand? Can you tell me what you are thinking about? Can you tell me what you mean? Tell me more.*
2. Grappling with a contradiction: *What do you* think*? What is happening here?*
3. Activating Prior Knowledge: *How would you use . . . to . . . ? What is a new example of . . . ? How is . . . related to . . . that we studied earlier?*
4. Revoicing: *What I hear you say is . . .*
5. Wait time: *Do you want more time to think?*
6. Modeling: *I think I'll see what's going to happen if I use your strategy for . . . , ok? Let me see, how about trying . . . ?*

"Modeling" is helpful if a student or group is completely stuck or attached to a strategy that will not work out.

3. General Moves for Emotional Support Doing inquiry-based mathematics can be new and unsettling to students. Supporting them emotionally is often as important as supporting them mathematically. If they are unmotivated it can help to draw them in by showing them what *you* think is interesting—maybe using a video, a drawing, a pattern, showing a connection with history or an application, etc. We are also using tools from the Nurtured Heart Approach. This approach, among other things, uses positive reinforcement to encourage persistence, sharing, curiosity, creativity, etc.

1. Student is insecure and fishing for evaluation: *I am not saying that you are right or wrong; I am just wondering. / What do* you *think?*
2. Student is frustrated: *I know it's hard. / I love your persistence, I can see that you are not giving up. / Why don't you check in with . . . ?*
3. Student is anxious: *You can do it! / Take a breath. / Get coffee. / Look back at . . . I know you could do this before. / What would help you right now?*
4. Student is unmotivated or unfocused: *Which problem are you thinking about? / Let me show you something cool (maybe unrelated). / Back to math / Let's focus. / I think this is so interesting because . . .*

4. Ending a Conversation Often we just walk away when we believe a student is now capable of continuing their quest. This conveys to the students that you trust them to figure it out for themselves. The conversation usually does not end with the solution of the problem, since then we have probably "given too much away".

1. Walk away: *You can do this, I'll check in later.*
2. Walk away silently (if students are engaged in their work).